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ORBIT SELECTION STUDY FOR PAGEOS

SATELLITE FINAL REPORT

Contract No. NAs 1-4614

By S. Worley, J. Miller, and G. Linsenmayer

Westinghouse Electric Corporation
Aerospace Division
Baltimore, Maryland

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

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An investigation was made to select a suitable orbit for the PAGEOS Geodetic Satellite program. Selection was based on obtaining a maximum number of satisfactory observation opportunities, subject to certain initial orbit constraints. The final report contains a complete discussion of all factors involved in the selection, and a discussion of the characteristics of the selected orbit.

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INTRODUCTION

The PAGEOS program is a program for measuring the shape of the earth by photographing an Echo satellite against the starfield background. Photographs are to be taken from camera stations in a network of approximately 35 or 36 stations which are distributed more or less evenly over the earth's surface. (Table I lists the station locations considered.) The direction of a camera/satellite sightline at the time of exposure is determined by interpolating the images of the satellite into the celestial coordinate system of the starfield background, with appropriate coordinate system rotation as determined by the sidereal time of the exposure. Simultaneous observation of a satellite from two adjacent camera stations determines a plane containing the baseline connecting them, and a second set of simultaneous observations determines a second plane whose intersection with the first gives the direction of the baseline. Other baselines are similarly found and are combined in an appropriate adjustment to give the shape (except for scale factor) of the observation station network. Observations may be made simultaneously from two or three stations; three-station observations are to be preferred since information is gained relating to all three baselines in the observation station triangle, whereas two-station observations yield data only relating to one baseline.

To be practical and suitable for such a program, the orbit of the satellite must satisfy a number of conditions. These are:

   a.   The orbit must initially be totally sunlit for a period of 14 days to ensure proper inflation of the Echo satellite.

   b.   The launch window must be at least 1 hour.

   c.   The initial apogee altitude must be in the range of 4000 to 4500 kilometers.

   d.   The initial eccentricity must be in the range 0 to 0.04.

   e.   The inclination must be in the range 80 to 90 degrees.

TABLE I

STATION LOCATIONS

No.	Station Name	Latitude (degrees)	Longitude (degrees)
1	Greenland, Thule AFB	76.5 N	68.7 W
2	U.S.A, Aberdeen, Md.	39.5 N	76.1 W
3	U.S.A, Larson AFB, Wash.	47.2 N	119.3 W
4	U.S.A, Aleutian Is., Shemya I.	52.7 N	174.1 E
5	U.S.S.R, Tura, Siberia	64.8 N	101.0 E
6	Finland, Kuopio	62.7 N	28.0 E
7	Azores Is., Pico I.	39.0 N	28.5 W
8	Dutch Guiana, Paramaribo	05.5 N	55.2 W
9	Equador, Quito	00.1 S	78.5 W
10	Clipperton I.	10.3 N	109.2 W
11	U.S.A, Hilo, Hawaii	19.8 N	155.0 W
12	Wake Island	19.7 N	166.2 E
13	Japan, Kagoshima	31.7 N	130.6 E
14	India, Gauhati	26.2 N	91.7 E
15	Iran, Sabzawar	36.5 N	57.5 E
16	Libya, Syrte	31.7 N	16.4 E
17	Liberia, Roberts Field	06.8 N	10.2 W
18	Trindade Island	20.5 S	29.4 W
19	Argentina, Villa Dolores	32.0 S	65.1 W
20	Sala y Gomez Island	26.6 S	105.2 W
21	Pukapuka Island	14.7 S	138.8 W
22	Wallis Is., Uvea I.	13.2 S	176.3 W
23	New Guinea, Kikori	07.3 S	144.2 E
24	Sumatra, Palembang	03.0 S	105.0 E
25	Maldive Is., Male'	04.2 N	73.3 E
26	Sudan, Juba	04.8 N	31.6 E
27	Southwest Africa, Bogenfels	27.8 S	15.8 E
28	So. Sandwich Is., Saunders I.	58.4 S	26.7 W
29	Antarctica, Peter I.	69.2 S	90.0 W
30	So. Pacific Ocean, Shoal	41.5 S	148.6 W
31	New Zealand, Queenstown	45.0 S	168.2 E
32	Australia, Denmark	35.0 S	117.3 E
33	St. Paul Island	38.7 S	77.0 E
34	Madagascar, Fort Dauphin	25.0 S	47.1 E
35	Antarctica, U.S.S.R Station	68.0 S	46.4 E
36	Antarctica, France Station	67.0 S	139.0 E

f. The argument of the perigee may be in the range 0 to 360 degrees.

g. The launch interval may be the entire year.

h. The lifetime of the orbit must be at least 5 years.

i. The orbit must provide a large number of suitable two-station and three-station observation opportunities.

The purpose of the orbit selection study was to choose, within the constraints of the first seven conditions, an orbit suitably satisfying the last two.

This choice is complicated by the fact that the effects of solar pressure on a high area/mass ratio satellite of the echo type lead to complex variations in eccentricity, inclination, and period. These variations are difficult to relate to the frequency of acceptable viewing opportunities without extensive computation. In this report, a complete discussion is presented of all factors considered in the selection of a suitable orbit. The general approach is described first, followed by a description of the computer programs used to aid in the selection. Finally, the numerical results obtained are discussed and the selected orbit is specified, with reasons for its selection. Further detailed information regarding the computer programs and a preliminary graphical analysis may be found in the appendixes.

The criteria employed to judge the acceptability of a viewing opportunity were:

a. The viewing station must be in darkness (i.e., the sun is at least 18 degrees over the horizon) while the satellite is sunlit.

b. The elevation angle from the station to the satellite must be at least 30 degrees. (Elevation angles of 25 degrees are considered as marginal.)

c. The sun-satellite-camera included angle must not exceed 135 degrees.

d. The viewing conditions must be acceptable at two or three stations simultaneously, for a period of at least 2 minutes.

e. The satellite must not exceed an altitude which would seriously degrade the resolution of the photograph (i.e., 5000 km).

f. To complete the observations for each baseline, at least two observations must be made to define planes intersecting the baseline at an angle of 60 degrees or greater.

APPROACH

Several computer programs were used to aid in the orbit selection process. The use of these programs is illustrated by the block diagram of figure 1 of the following section and is explained in complete detail in that section. Running time proved to be a severe problem in the single-station observation program, so that the following approach to the selection of an orbit was chosen to permit efficient use of computer time.

The first step in the selection process was to examine a number of possible orbits (i.e., orbits satisfying the initial launch restrictions) using the Lifetime - 18 program. Of these orbits, those having poor lifetimes, large eccentricity variations, or other objectionable characteristics were rejected, and six of the most promising orbits were chosen for further study.

To obtain a further comparison of these six orbits, an abbreviated problem was defined. In this problem, a network of seven representative stations (stations 8, 17, 18, 27, 28, 29, and 35) were chosen from the complete network. The numbers of viewing opportunities were found for this abbreviated network by using the single-station observation program and the simultaneous observation program; however, these calculations were performed only for 1 month out of 3, over a period of 3 years. It was believed that such a sampling would provide a suitable comparison of the orbits. A tentative comparison and selection were made by tabulating the results and selecting the orbit which gave the best overall performance, as explained in detail in the section entitled "Discussion and Results."

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A complete 5-year run was then made using the full 36-station network to ensure that the selected orbit was satisfactory for the complete problem. In addition, this run was expected to provide a check of the assumption that the abbreviated problem was a reasonable approximation to the full problem. Finally, using Lifetime-18, variations within the launch window of the selected orbit were studied to determine their effect on lifetime, eccentricity changes, etc.

COMPUTER PROGRAMS

Digital Computer Programs and Interfaces

The philosophy which has been adopted in approaching the problem of selecting an orbit for the PAGEOS (PAssive GEOdetic Satellite) is that of simulating the orbit and the available observations from the system of ground stations by a series of digital computer programs. Essentially these programs consider the occurrence of simultaneous observations by two or three ground stations from a satellite in a continuously perturbed orbit. Figure 1 shows a block diagram of the digital computer programs and their respective interface exchanges of information.

The first program shown is known as the "Lifetime 18" program and was originally developed by the NASA Goddard Space Flight Center. This program has been modified extensively in the area of earth shadow determination, and additions have been made to calculate other quantities significant to this study. "Lifetime 18" is essentially an orbital prediction program which uses a variation of parameters like method of integrating the long term orbital perturbations over a constant step size of one day to determine a time history of each of the orbital elements. Among the perturbing forces considered are those due to the second, third, and fourth harmonics of the earth's gravitational field, the direct solar pressure forces on a uniformly

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┌─────────────────────────────┐              ┌─────────────────────────────┐
│      "LIFETIME 18"          │              │     LOGISTICS PROGRAM       │
│ CALCULATE ORBITAL PERTURBATIONS │          │ DETERMINE AN OPTIMUM TIMETABLE │
│ OVER TIME PERIOD OF INTEREST │              │ FOR CAMERA MOVEMENTS BASED  │
│                             │              │ ON WEATHER, TRAVEL COSTS AND│
│                             │              │ OBSERVATION PROBABILITY     │
└─────────────────────────────┘              └─────────────────────────────┘
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ORBITAL PARAMETERS;
ORBITAL ELEMENTS,
EARTH SHADOW ANGLES, ETC.
AS A FUNCTION OF TIME

END OF PHASE II

FREQUENCY OF OBSERVATION TABLE:
TABLE OF FREQUENCIES OF OBSERVATION
AS A FUNCTION OF TIME FOR EACH PAIR
AND TRIANGLE OF STATION

SINGLE STATION OBSERVATION PROGRAM
SOLVE FOR TRANSIT TIMES OVER
EACH STATION USING SINGLE
STATION RESTRICTIONS

SIMULTANEOUS OBSERVATION PROGRAM
SORT TRANSIT TIMETABLE
ACCORDING TO TIME AND
DETERMINE SIMULTANEOUS OBSERVATIONS
FROM ADJACENT OVERLAPPING TIMES.
APPLY GENERALIZED ALTITUDE TEST IN
THIS PROGRAM

TRANSIT TIME TABLE:
TRANSIT TIMES AND OTHER RELATED QUANTITIES

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Figure 1.   Block Diagram of Digital Computer Program

coated satellite, the gravitational forces of both the sun and the moon, and the effects of atmospheric drag on the satellite. This program is used to generate a time history of the orbital elements of the satellite which are stored on magnetic tape for use by the next program.

The second program in the chain is called the "Single Station Observation Program" and has been developed for this study. It uses the orbital parameter information generated by the "Lifetime 18" program and solves for the times when the satellite will pass over each station. All of the single station restrictions, such as the requirements that the station be in darkness, the satellite be in sunlight, and the elevation angle exceed a minimum of 30 degrees, are used in this program. The application of these restrictions and the methods of solution are explained in much greater detail in the next section. The program generates, as output, a magnetic tape containing the observation time limits for each station and the radius vector to the satellite at the time of entrance to and exit from an observation.

The third program in the chain is called the "Simultaneous Observation Program" and it also has been developed for this study. This program is described in more detail in a later section. Basically, it chronologically sorts the table of transit times obtained from the "Single Station Observation Program" and determines the simultaneous observations from adjacent overlapping times in the sorted table. It also applies a generalized altitude test (as a function of baseline distance) and calculates the angle which the satellite makes with respect to a vertical plane through the baseline. As output, it generates a frequency of observation table which provides the number of observations available over a specified time period for each defined pair and triangle of stations.

This frequency of observation table has been used as the basis for selecting the orbit, thus marking the completion of this study. It is also to be used as an input to a subsequent study which will determine a camera logistics

strategy to be used to optimize the time required to complete all the observations.

A fourth program, called the "Logistics Program," will be developed for this purpose. It will consider, along with the frequency of observation table, the probability of cloud coverage over each station and the travel times between each pair of stations to determine an optimum timetable of camera movements.

Single Station Observation Program

The second program in the set of three used in the determination of viewing opportunities for various orbits is the "Single Station Observation Program." It uses the orbital parameter data generated by the "Lifetime 18" program to determine the successful transit times of the satellite over each of the ground stations. Among the restrictions considered in determining the successful transit are that the station be in darkness (sun at least 18 degrees below horizon), the satellite be in sunlight, the satellite pass through a 60-degree cone of observation around the ground station (marginal observations are considered down to 65 degrees), and these conditions prevail for at least 2 minutes.

It has been found that the restriction imposed on the station, satellite, sun angle (must be less than 135 degrees) can only be exceeded occasionaly for marginal observations and never for elevation angles above 27 degrees. The geometry of this situation is shown in appendix I. Since this restriction can never be exceeded for a good observation, it is not considered in the "Single Station Observation Program."

The transit times calculated by this program are passed along by means of magnetic tape to the "Simultaneous Observation Program" where they are compared on a time basis to determine simultaneous observations. The altitude restriction (5000 km.) is also considered in this program.

A macro diagram of the program is shown in figures 2 and 3. This diagram is by no means meant to show the detailed implementation of the program, but is intended to show the order in which the computations are made and the logic involved in carrying out the tests on the various restrictions.

The program essentially consists of three loops. The inner loop iteration is performed over each of the stations, the middle loop is performed over each orbit, and the outer loop is performed over each day.

As shown in figure 1, the program is started with an initialization process and immediately passes into the block which reads the orbital parameter information produced by the "Lifetime 18" program. This information consists of the following orbital parameters: a (semimajor axis), e (eccentricity), i (inclination), Ω (right ascension), ω (argument of perigee), and the eccentric anomalies at which the satellite passes into and out of the earth's shadow. Immediately following this the sun vector, which is assumed constant over a day, is calculated for that day as described in appendix II. The station darkness times, i.e., the times at which each station enters and leaves the region of total darkness for that day, are calculated as described in appendix III. This completes the computations which must be made only once every day and the program passes into the test for the end of day.

The computations which must be performed only once an orbit are shown in the block following the end-of-day test. These consist of updating the end-of-orbit time to the next right ascension crossing time and calculating the times during which the satellite is in the earth's shadow. Successive right ascension crossings are, of course, separated by increments of 2π on the angle $u + \omega$ (true anomaly plus argument of perigee). The times at which this angle is an integral multiple of 2π define the points of right ascension crossing. The times of right ascension crossing are then calculated by solving the equation:

$$\sin (u + \omega) = 0$$
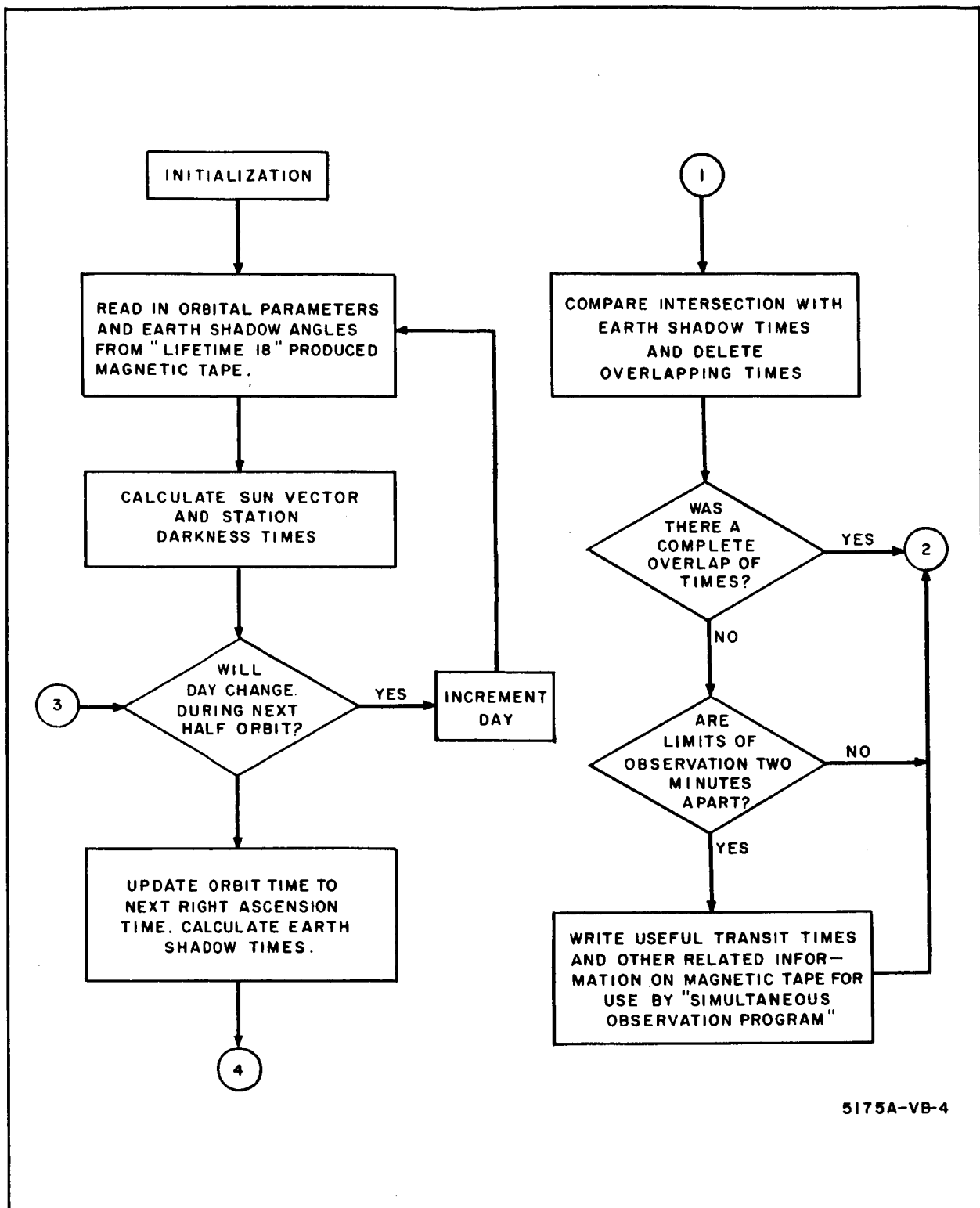
Figure 2. Single Station Observation Program

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Figure 3. Single Station Observation Program

5175A-VB-5

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Due to the variations of the orbital parameters, the orbit does not close upon itself and hence this equation cannot be solved explicitly. For this reason an iteration is performed in the region of an orbital period away from the present right ascension crossing to find the time which satisfies the equation. The true anomaly and argument of perigee are related to time in the manner described in appendix IV.

The earth shadow times are calculated from Kepler's equations and are:

$$M = E - e \sin E$$

$$t_s = T_o + \frac{M}{n}$$

where E is the eccentric anomaly of the satellite's passage into the earth's shadow obtained from the "Lifetime 18" information, T_o is the time of perigee passage, and n is the mean angular motion of the satellite. The computation is, of course, repeated for the angle of exit from the earth's shadow.

The information shown in figure 3 and on the right-hand side of figure 2 is all contained in the station loop of the program and shows the coarse tests and logic involved in determining the successful observations. Since the calculations of the times of entrance into and exit from the cone of observation as described in appendix IV take a relatively large amount of machine time, the philosophy which has been adopted is to reject as many stations as possible on coarse tests before performing the more detailed calculations. The first test shown upon entering the station loop in figure 2 is such a test; i.e., if the station is not in darkness at any time during the current orbit, the program passes immediately to the next station. The coarse elevation test, described in appendix V and shown in figure 2, is another such test and checks to see whether the elevation angle can possibly be greater than the allowable minimum.

Prior to this the point of maximum elevation had been estimated by the procedure described in appendix VI. If the coarse elevation test is passed, this estimation is used as a starting value for calculating the maximum elevation time as described in appendix IV. If the resulting maximum elevation is greater than the allowable minimum (25 or 30 degrees), the program calculates either one or both of the minimum elevation times, depending on the conditions of station darkness in that region of time. This procedure is also described in appendix IV.

After the limits of observation have been obtained with respect to the cone of observation and the station darkness times, they are compared with the earth shadow times, and overlapping times are deleted as shown in figure 1. A final test is made to determine if the resulting limits of observation are 2 minutes apart. If all these test requirements are met, the limits of observation and the radius vectors to the satellite at the times of entrance to and exit from the observation are written on a magnetic tape to be used by the "Simultaneous Observation Program." The radius vectors are used both in making the altitude test and in calculating the angle which the satellite makes with respect to a vertical plane through the baseline.

When the computations for a particular station have been completed, either through failure to pass one of the tests or through the calculation of a successful observation, the next maximum elevation time is estimated and the program either processes that time or goes on to the next station if that time is in the current orbit.

For the system of 36 ground stations currently being studied, the program will run at a rate of approximately 2 days of real time to 1 minute of UNIVAC 1107 machine time and 10 days of real time to 1 minute of IBM 7094 MOD II machine time.

Simultaneous Observation Program

The "Simultaneous Observation Program" determines (from a given set of single station observations) those ground stations which simultaneously observe the satellite for a given time interval. The program tabulates for each station pair the specific data concerned with that observation; e. g., marginal or acceptable observation, acceptable satellite altitude, entrance and exit line of sight angles, etc. These data are accumulated and printed at a given interval as well as tabulated according to time of occurrence. The program also searches for simultaneous observations among any three of the given ground stations. The data concerned with the "simultaneous triangle observations" is tabulated according to time of occurrence as well as accumulated and printed periodically. The program has a complete battery of diagnostics including information such as those station pairs and triangles which were not defined as such, those station pairs which simultaneously observed the satellite but at an unexceptable altitude, etc.

A simplified flow diagram for the simultaneous observation program is found in figures 4 and 5. Basically, the program works as follows. A set of single station observations is read in from a magnetic tape which was previously generated by the "Single Station Observation Program." These data consist of the station number, times of entrance into both the 25- and 30-degree cones of observation, times of exit from both the 25- and 30-degree cones of observation, and the radius vectors to the satellite at the 30-degree entrance and exit times.

This set of data is then sorted on the 30-degree entrance time for each single station observation. The program then steps through this set of single station observations — first looking two observations ahead in order to search for simultaneous observations between three ground stations and then, if a triangle is not encountered, looking ahead one observation for a simultaneous observation between a pair of stations. The criterion for a
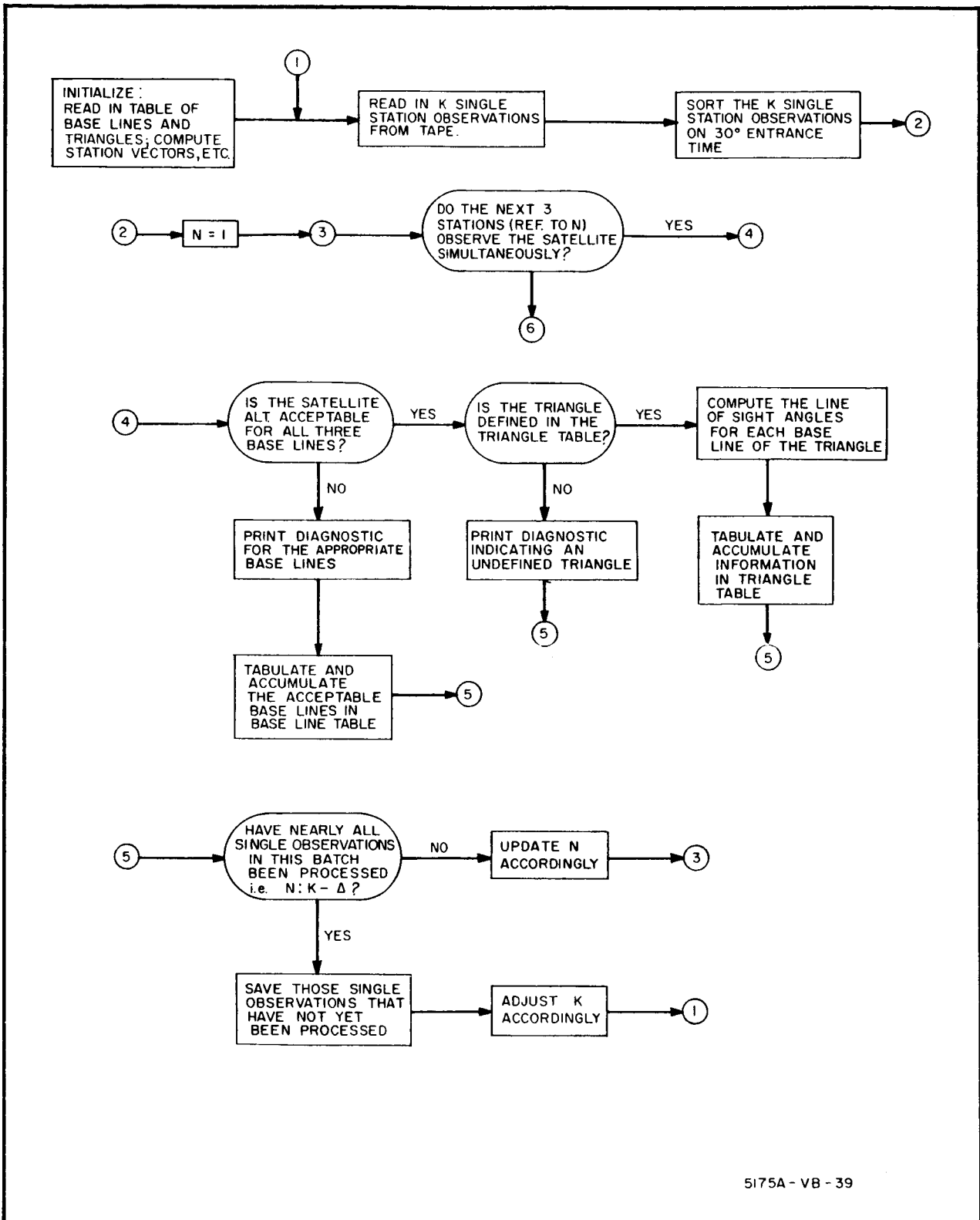
Figure 4. Simultaneous Observation Program
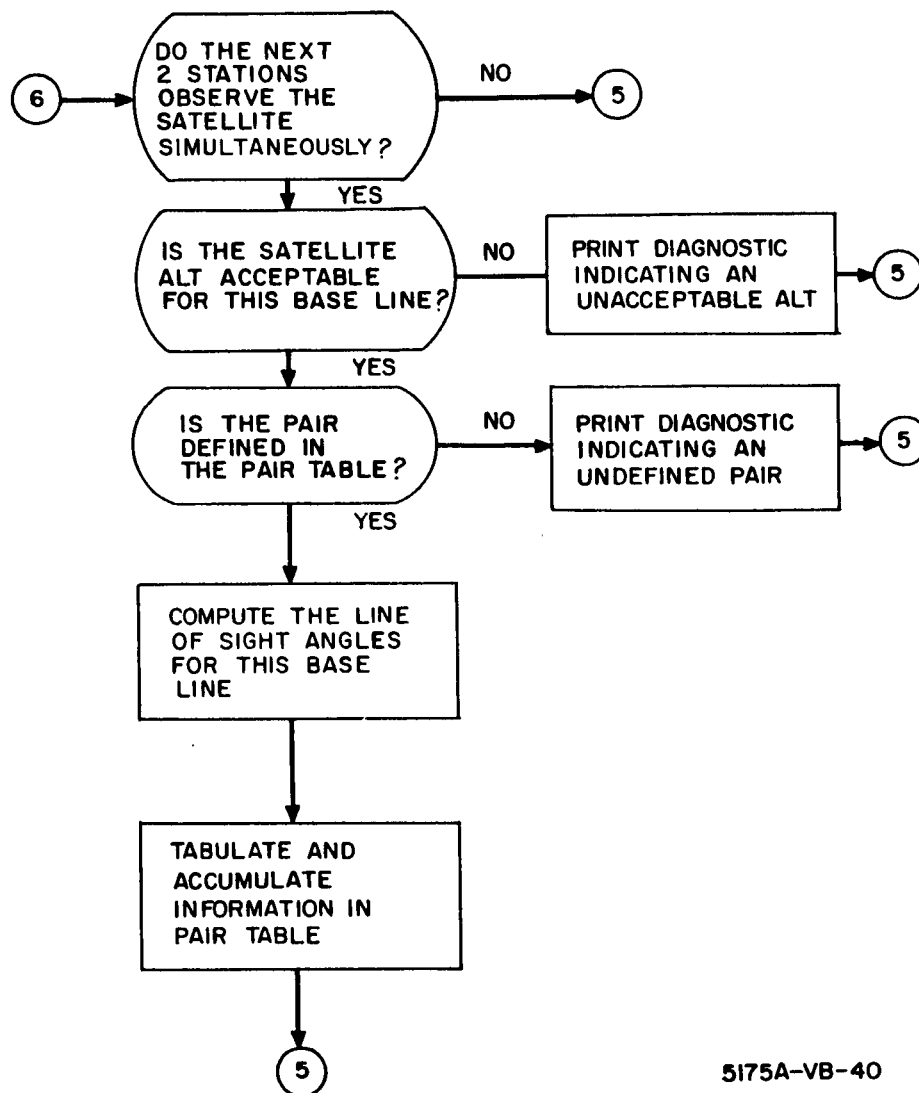
Figure 5. Simultaneous Observation Program

simultaneous observation is a 2-minute time interval for which both ground stations observe the satellite. This interval is determined by examining the entrance and exit times of both stations, determining the overlapping time interval, and comparing this with 2 minutes.

If the stations are found to observe the satellite simultaneously for a period of 2 minutes, the observation is considered acceptable if both the entrance and exit times are the 30-degree times. The simultaneous observation is considered marginal if the 25-degree entrance or exit time is used for either station. For all simultaneous observations between stations, the altitude of the satellite is checked. If the altitude of the satellite is computed to be greater than 5000 km., a diagnostic is printed and the station pair is not recorded as acceptable. Also, for a simultaneous observation between two stations the line-of-sight angle between a plane through the station pair baseline and the center of the earth and a plane coincidental with the satellite and the station pair baseline is computed for both the entrance time and the exit time. The method used to determine these angles may be found in appendix VII.

Finally, all simultaneous observations for station pairs and station triangles are checked to see if they are in a table of previously defined pairs and triangles. If these observations are found to be in the table, the information associated with the observation is stored in the baseline or triangle table. At a specified print interval the information is accumulated and printed. If the station pair or triangle is not defined in the appropriate table, a diagnostic is printed. Finally, it should be noted that there are various options in the program concerned with printing and running the job in a piecewise fashion.

DISCUSSION AND RESULTS

Initial Orbit Selection

Due to the impracticality of running all orbits through the complete chain of programs (because of the machine time problem on the "Single Observation Program"), a preliminary set of orbits has been chosen based upon the results of the previous orbital stability study.*

The first orbit selected is that tentatively chosen by the NASA Langley Research Center from the results of the previously completed study and is a circular orbit at 87 degrees inclination at an altitude of 4250 km. From a probable launch date of late second quarter, 1966, the launch date of June 1, 1966 was chosen. In addition, the high inclination orbit (greater than 85 degrees) which produced the least eccentricity at each altitude was chosen. This provided orbits of 85 degrees at 4000 km, 89 degrees at 4250 km, and 90 degrees at 4500 km. One further orbit was selected, that being the orbit which produced the least eccentricity of all, the 80-degree orbit at 4500 km.

Since it had been shown in the orbital stability study that the maximum eccentricity reached by an orbit was also dependent on the time of day of launch, these five orbits have been studied extensively with respect to the launch time of day through the use of the "Lifetime 18" program. The results of this study are shown in figures 6 through 10.

As can be seen, the launch time of day (launch right ascension) is very critical to the maximum eccentricity that the orbit reached. In all cases except the 4500-km, 80-degree orbit, the satellite will have a lifetime of less than 5 years for a launch during certain critical portions of the day.

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*"Stability of Orbital Parameters of an Echo I Type Satellite," Final Report for Phase III of NASA contract NAS 1-3131, July 1964.
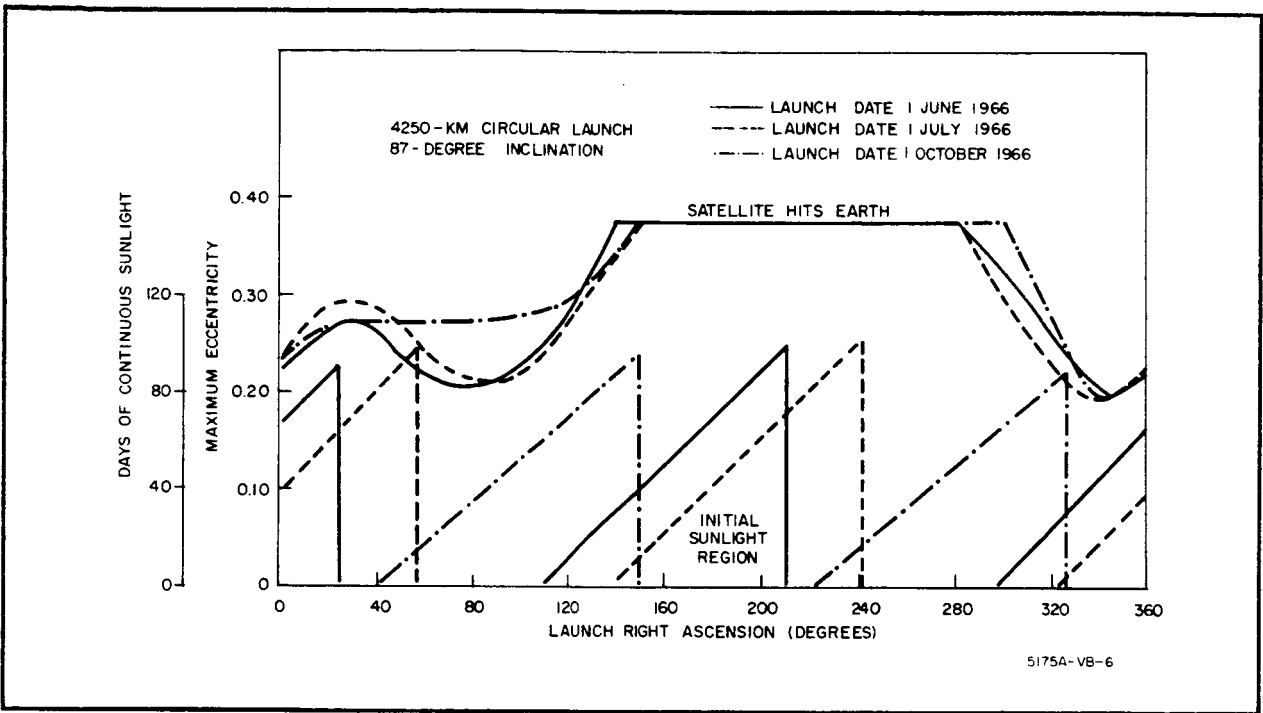
Figure 6. Maximum Eccentricity vs Launch Right Ascension - Orbit 1
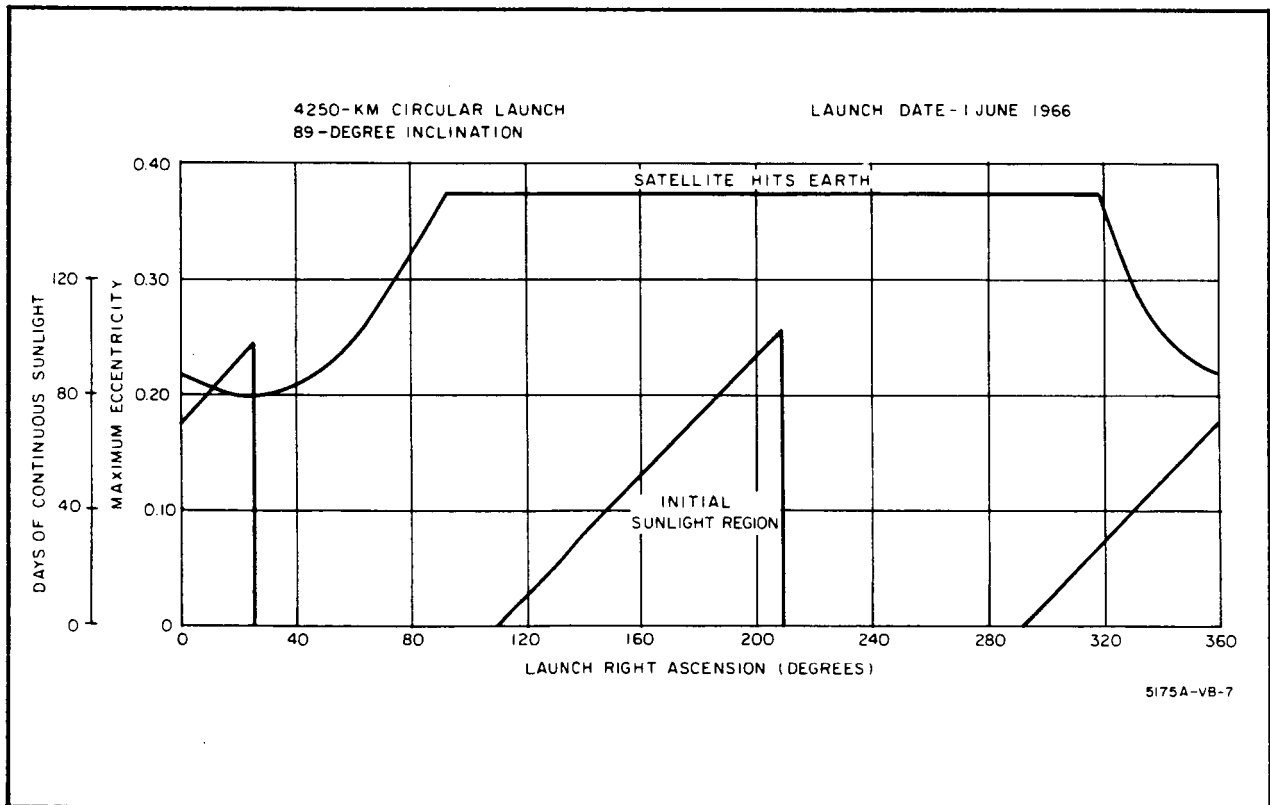
Figure 7. Maximum Eccentricity vs Launch Right Ascension - Orbit 2

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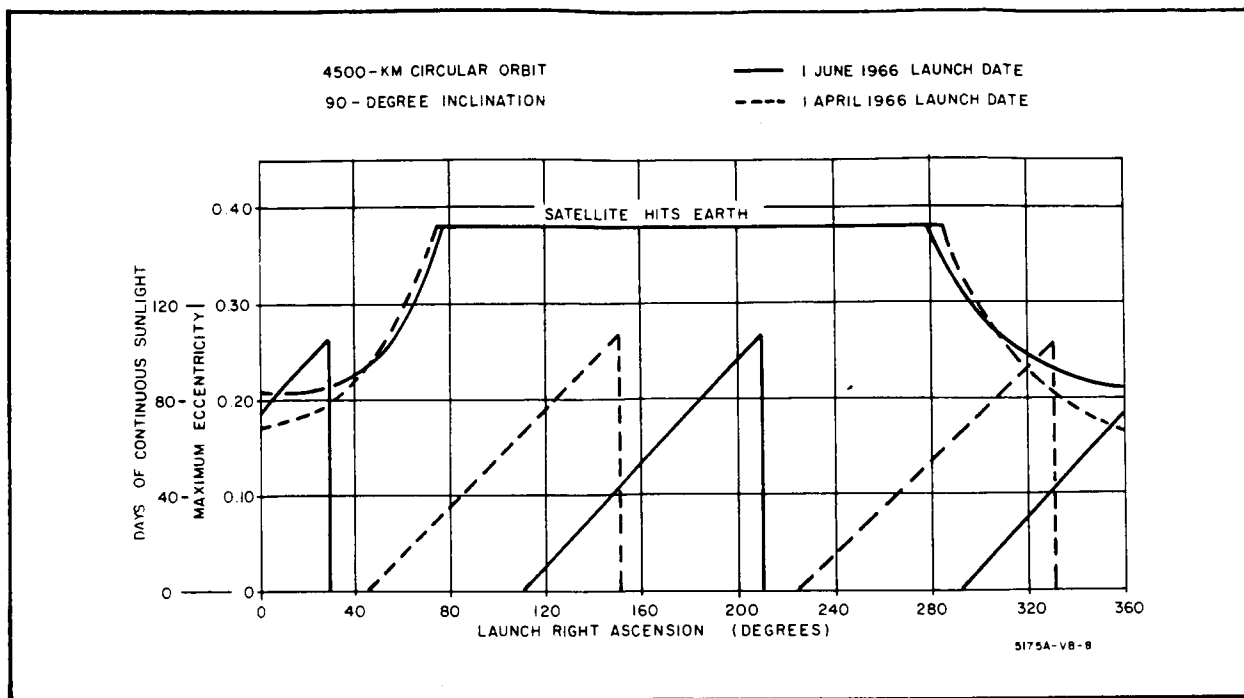
Figure 8.   Maximum Eccentricity vs Launch Right Ascension - Orbit 3
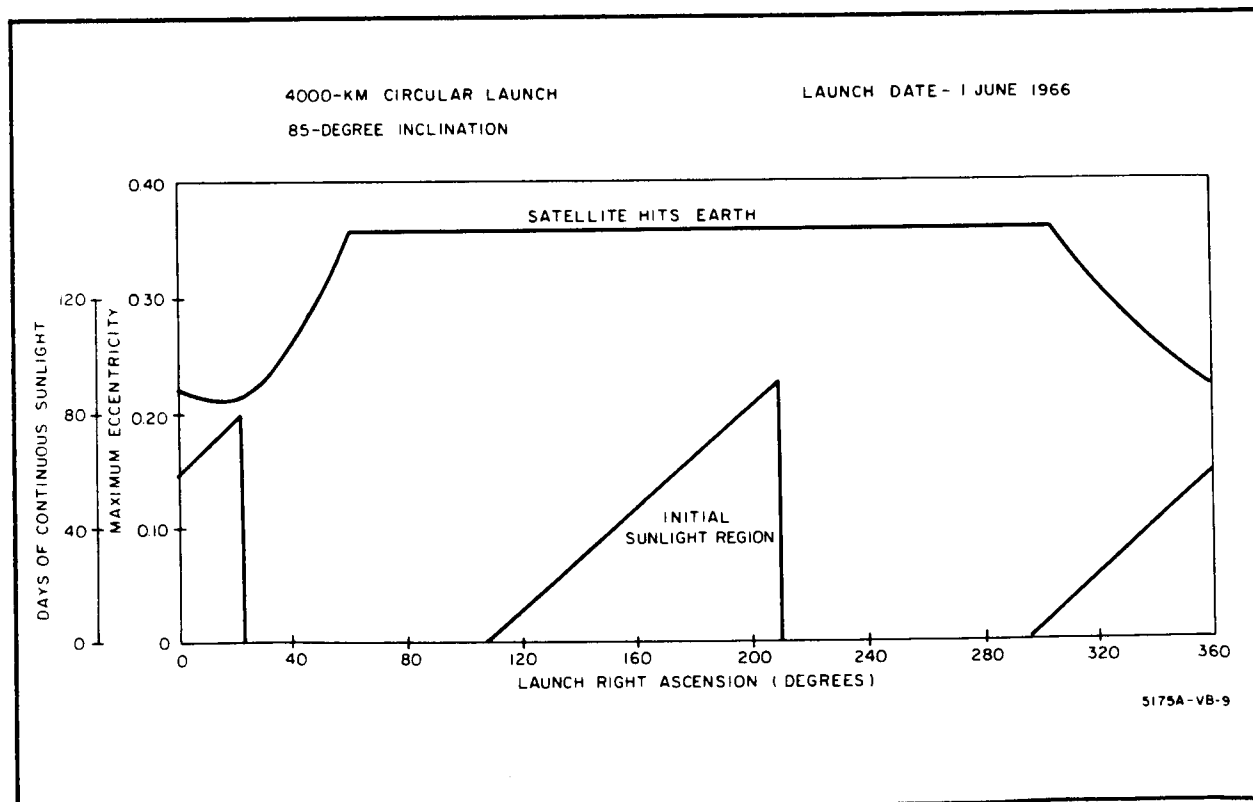
Figure 9.   Maximum Eccentricity vs Launch Right Ascension - Orbit 4

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4500-KM CIRCULAR LAUNCH   LAUNCH DATE - I JUNE 1966
80-DEGREE INCLINATION

DAYS OF CONTINUOUS SUNLIGHT

MAXIMUM ECCENTRICITY

INITIAL SUNLIGHT REGION

LAUNCH RIGHT ASCENSION (DEGREES)

5175A-VB-10
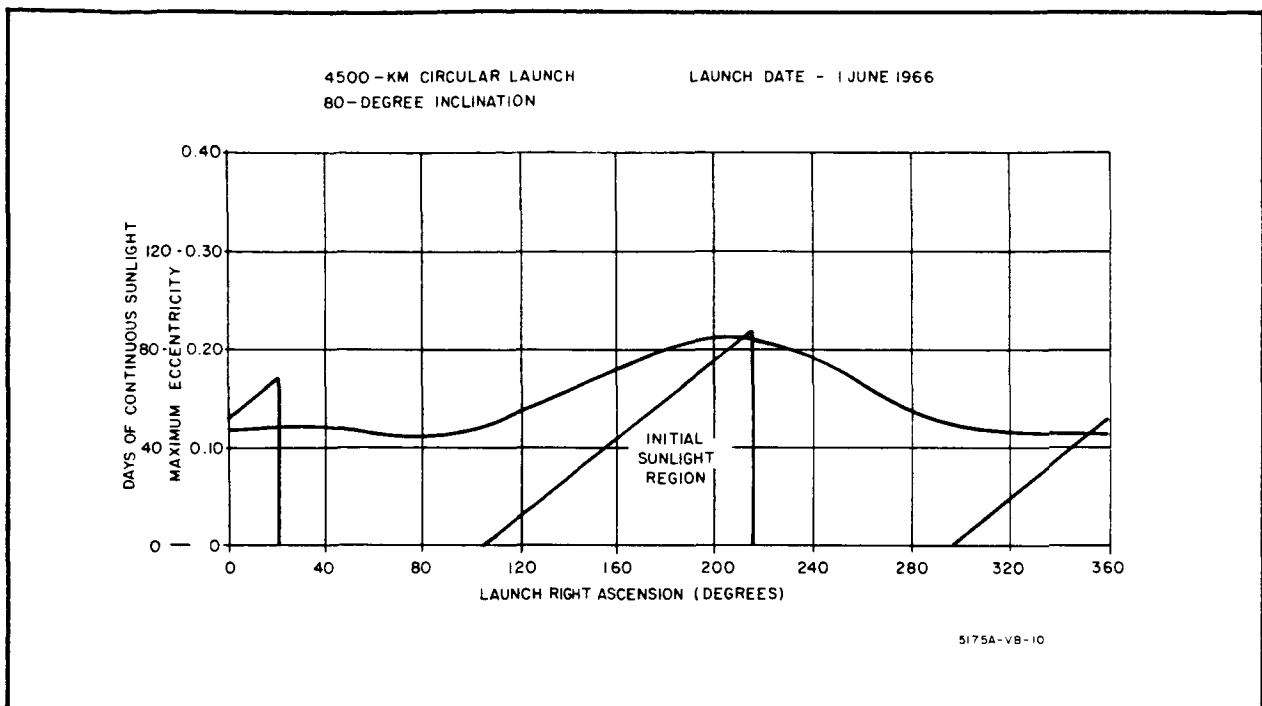
Figure 10.   Maximum Eccentricity vs Launch Right Ascension - Orbit 5

However, the minimum launch window of 1 hour (15 degrees of right ascension) is sufficient in all cases to insure the obtaining of new optimum maximum eccentricity for that orbit.

Figures 6 and 8 show the effects of different launch dates on both the maximum eccentricity and the initial periods of continuous sunlight. As can be seen, the maximum eccentricity curves retain their same general shape, but in some cases reach different values of maximum eccentricity. As would be expected, the initial continuous sunlight regions simply slide along the launch right ascension axis.

In choosing the launch right ascension for the orbits to be studied further, the criterion used was that it should produce a minimum eccentricity while fulfilling the condition of having initially at least 14 days of continuous sunlight. The orbits chosen for further study can then be summarized as follows.

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Orbit No.	Altitude (km.)	Inclination (deg.)	Launch Right Ascension (deg.)	Launch Date
1	4250	87	345	1 June 1966
2	4250	89	10	1 June 1966
3	4500	90	10	1 June 1966
4	4000	85	10	1 June 1966
5	4500	80	0	1 June 1966
6	4250	87	80	1 Oct. 1966

In summary, orbits 1 through 4 were chosen because of their combination of low eccentricity and high inclination, orbit 5 was chosen because of its extremely low eccentricity even though the low inclination is known to cause problems in fulfilling the observation requirements for the polar stations, and orbit 6 was chosen to show the effects of a different launch date on the frequency of observations.

Final Orbit Selection

The six chosen orbits were analyzed further using an abbreviated problem in which only seven stations were considered and in which the viewing opportunities were found for periods of 1 month every 3 months, over a time interval of 3 years. The seven stations of the abbreviated problem are listed in table II below. (The "old" numbers refer to the numbers of table I; the "new" numbers are those adopted for analysis of the abbreviated problem.) These particular stations were chosen because they formed a network which appeared to include a particularly representative selection of baselines and triangles.

TABLE II

SEVEN STATIONS OF THE ABBREVIATED PROBLEM

"Old" Number	Station	"New" Number
8	Dutch Guiana, Paramaribo	1
17	Liberia, Roberts Field	2
18	Trindade Island	3
27	Southwest Africa, Bogenfels	4
28	So. Sandwich Is., Saunders I.	5
29	Antarctica, Peter I.	6
35	Antarctica, U.S.S.R. Station	7

The locations of these stations (and also of the stations of the complete network) are shown on figure 11.

To evaluate the data obtained through consideration of the abbreviated problem, tables were prepared using results from the output of the simultaneous observation program.

Tables III and IV illustrate the results obtained for orbit 1 (which was the orbit finally selected). In this table, the tabulated entries are:

a. Number of good observations (G) in each time interval. (Time intervals were 1 month in duration at intervals of 3 months.)

b. Number of marginal observations (M) in each time interval.

c. Number of observations in which the angle of the observation plane is greater than 30 degrees from vertical (+).

d. Number of observations in which the angle of the observation plane is less than -30 degrees from vertical (-).

e. Totals for the 3-year period.

In table IV, the good (G) and marginal (M) entries apply to the whole triangle, while the (+) and (-) entries apply to the baselines whose endpoints are the 1st and 2nd, 1st and 3rd, and 2nd and 3rd stations listed, respectively.

TABLE III

BASELINE RESULTS

Baselines		607	698	789	880	970	1062	1153	1244	1335	1426	1517	End	Total
		END OF TIME INTERVAL IN DAYS FROM 1 JAN 1965												
1-2	G				3	1								4
	M		4		1						1			6
	+													
	−				2									2
1-3	G	3		1	3	6	10		1			11		35
	M				1	1		11				3		16
	+	2		1	1	2						2		8
	−						7		1			2		10
2-3	G	2		8	1	5	7				11	11	4	49
	M	1		4	1	7		2			3		1	19
	+	2		7		2					2			13
	−				2		5					5		12
2-4	G	3			16	2	15		4			9		49
	M	2			2	4	3	1	2		1	2		17
	+	1			2		4							7
	−				8		2		7					17
3-4	G				16			2				2		20
	M		1											1
	+				4									4
	−				16									16
3-5	G	21	22			6	14						22	85
	M	6	3			2	4				1		5	21
	+	11				1								12
	−	5	7				5						4	21
4-5	G	5	1		12								16	34
	M	2	1										2	5
	+	3											8	11
	−				9									9
4-1	G	2	21		16								2	41
	M	4	1		5		10							20
	+		2											2
	−													
5-6	G	43	12			38			2	35			9	139
	M		1			1				2				4
	+	60	4			27			4	17			2	114
	−	3	9			10							9	31
5-7	G	36	9			22				25			5	97
	M					2								2
	+													
	−	59	11			44				23			4	141
6-7	G	1				3								4
	M													
	+													
	−													
Total	G													557
	M													111
	+													
	−													

ORBIT 1:   4250 km, 87 degree INCLINATION
Ω = 345 degrees, 1 JUNE 1966 LAUNCH

Figure 11.   36- and 7- Station Network
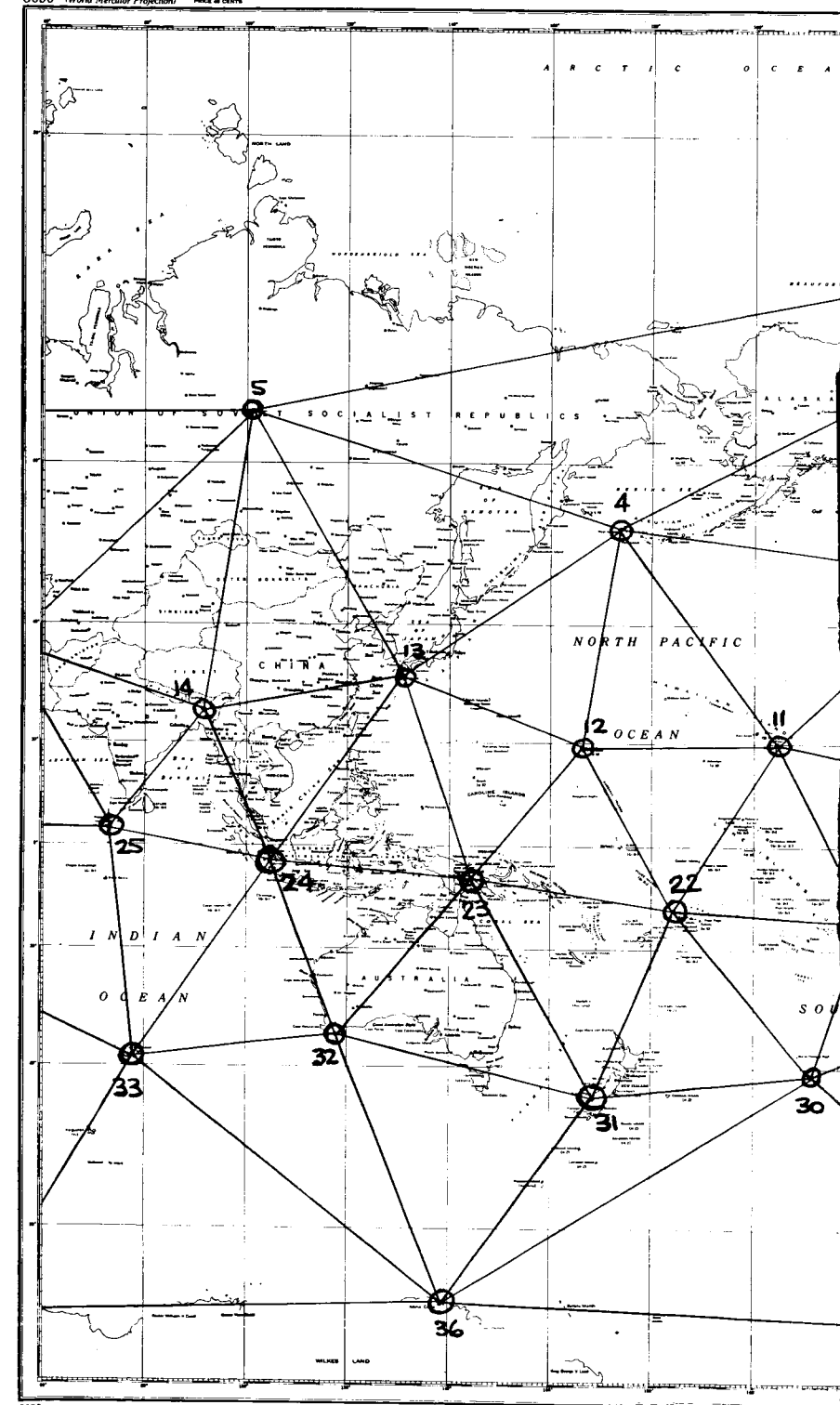
TABLE IV

TRIANGLE RESULTS

Triangles and Baselines		END OF TIME INTERVAL IN DAYS FROM 1 JAN 1965												
		607	698	789	880	970	1062	1153	1244	1335	1426	1517	End	Total
1 2	G				1		7							8
	M					1	1					1		3
	+				1		6							7
	-						3							3
3	+				1		7							8
	-													
	+													
	-				2		9							11
2 3	G													
	M				2							4		6
	+													
	-													
4	+													
	-													
	+													
	-													
3 4	G		5										1	6
	M		3										3	6
	+													
	-		5										1	6
5	+		4										2	6
	-													
	+													
	-		3											3
4 5	G	2	8										1	11
	M	4	9			4							6	23
	+	2	4										1	7
	-													
7	+													
	-		6											6
	+	2	8										1	11
	-	2	7										1	10
5 6	G	39				34				3				76
	M	1												1
	+	64				63				3				130
	-	1												1
7	+	3				8								11
	-	58				35				4				97
	+					4				1				5
	-													
Total	G													101
	M													39
	+													
	-													
	+													
	-													
	+													
	-													

Orbit 1:   4250 km, 87 degrees INCLINATION
Ω = 345 degrees, 1 JUNE 1966 LAUNCH

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For example, the (+), (-) entries associated with triangle 2-3-4 refer to baselines 2-3, 2-4, and 3-4 respectively. Negative entries are associated with observation planes which slant upward and to the right when the higher numbered station is viewed from the lower numbered station, positive entries with planes slanting to the left. These entries were tabulated for all baselines (table III), and for each triangle and all baselines of each triangle (table IV). Observations shown on table III are in addition to those of table IV, so that the total number of observations associated with any given baseline may be obtained by summing the information in the two tables. A lower bound on the number of observations intersecting at angles of 60 degrees or greater may be obtained by summing the (+) and (-) entries. (This is only a lower bound; for example, a 60-degree intersection could be obtained with observation plane angles of +40 and -20 degrees, but this latter observation would not be tabulated in the (-) category since it is greater than -30 degrees.)

Similar tables of appendix VIII show results obtained for other orbits.

A summary table was then prepared (table V), using data for all six orbits considered. The entries in this table are:

a. Total number of good/marginal observations for all baselines (excluding observations for triangles).

b. Number of observations for the most observed and the least observed baselines (including triangle observations).

c. Total number of good/marginal observations for all triangles.

d. Number of baselines for which observations were not obtained in both the (+) and the (-) categories.

e. Number of triangles and baselines unobserved.

Examination of table V reveals that, of the first 5 orbits, orbits 2 and 3 are relatively undesirable, due to a large number of "no (+) and (-)" occurrences and because the numbers of observations associated with the

TABLE V

PRELIMINARY ORBIT RESULTS

Orbit	Baselines			Triangles[1]	No (+) and (-)	Misses	
	Total[1]	Max[2]	Min[2,3]			Triangles	Baselines
1	559/111	217	12/26	103/39	1	1	0
2	590/102	222	1/5/6	80/38	5	1	0
3	460/127	124	3/10	55/39	6	1	0
4	535/101	207	5/27	127/38	0	0	0
5	601/84	162	10/40	107/48	1	0	0
6	662/108	116	13/14	44/24	0	0	0

Notes: 1 Good/Marginal

2 Triangles plus baselines, good observations only

3 Listed in order of number of observations, with minimum first

least observed baselines are rather low. Of the remaining 1 June orbits (numbers 1, 4, 5), orbit No. 1 was chosen for the following reasons:

a. Orbit 4 scores somewhat low in terms of total baseline observations and number of observations of the least observed baseline.

b. Preliminary graphical analysis (appendix IX) indicates the possibility of difficulties in obtaining satisfactory observation of all near-polar baselines with an orbit inclination of 80 degrees. Although this was not indicated during study of the abbreviated problem, it is possible that difficulties might arise with consideration of the complete network. For this reason, the more highly inclined orbit 1 appeared to be a more conservative choice.

In addition to the five orbits discussed above, a sixth run was made using the same altitude and inclination as used for orbit 1, but with a 1 October launch date. Comparing these results with those for 1 June launch, the 1 October launch is also seen to give good performance, having more total baseline observations and fewer triangle observations.

After selection of orbit 1 based on the considerations discussed above, a complete 5-year run was made for this orbit. In this run, the 36-station network was used and time sampling was not employed. The results of this run, presented in the form of tables similar to tables III and IV, may be found in appendix X. In summarizing these results, several points were noted:

a. All required observation conditions were fulfilled. Simultaneous three station of observations were obtained at all triangles except the polar triangles. However, the baselines of the polar triangles were individually observed as required.

b. A definite pattern of observation opportunities at high northern and southern latitudes was observed. Observations were poor at high northern latitudes during the summer months (June, July, August) and good during winter months, with the conditions at southern latitudes exactly

reversed. This was expected, due to the variation in the lengths of day and night as a function of season.

c. Observations were more common during the first year than during later years. This is due to the fact that the orbit is initially circular, allowing observations at all points of the orbit; however, in later years, the orbit becomes eccentric until portions of the orbit exceed the 5000-km-altitude limitation, reducing the number of observation opportunities possible.

d. The results of the 5-year run agreed reasonably well with those of the corresponding sampled run. This point is discussed more fully at the end of this section.

Time of Day of Launch

To determine the exact time of launch required to achieve a given right ascension, it is necessary to consider the exact launch trajectory from launch to orbit injection. For the 1 June launch, an approximate launch time was calculated ignoring the launch trajectory; the procedure of this calculation may be found in appendix XI. The approximate launch time was 6:30 AM, Pacific Standard Time, for launch 1 June 1966.

(24/n) Hour Orbits

If the period of a satellite orbit is an integral fraction of a sidereal day, the satellite would appear at the same position at the same time each day, resulting in a poor pattern of coverage. The altitude of the selected orbit is quite close to the altitude of such a resonant orbit, that altitude being 4160 km for a 3-hour orbit. However, even a small variation in altitude provides a satisfactory degree of nonresonance. For an altitude of 4250 km, the earth position under the satellite shifts about 3 degrees per day; thus, the total earth surface is covered in 15 days. (The orbit period is about 3 hours, corresponding to 8 revolutions per day or 45 degrees of earth revolution per orbit; 45 degrees/3 degrees per day = 15 days.)

During the 5-year period, large variations in orbit elements are encountered, as much as 60 km in altitude and 3 degrees in inclination. Because of these variations, even if the orbit initially has a 3-hour period, this condition would not long persist and a satisfactory degree of nonresonance would be attained. It was concluded from these considerations that resonant conditions as described above will not exist for any significant length of time and will not be an important problem.

Stability of Selected Orbit

With the nominal launch conditions chosen to be a 4250-kilometer circular orbit at 87-degree inclination, 345-degree right ascension and a date of 1 June 1966, a study was made to determine the orbital stability of the satellite as a function of deviations from the nominal conditions. The Lifetime 18 orbital prediction program was used to determine the orbital stabilities in this study. The time variations of the orbital elements of the nominal orbit have also been plotted in detail and are shown later in this section.

In figures 12 through 15 are shown the variations of maximum eccentricity and the initial period of continuous sunlight with respect to the launch right ascension (time of day). This information is given for 12 launch dates throughout the year (first day of each month) assuming a nominal conditions on the launch altitude, eccentricity, and inclination. As can be seen, the maximum eccentricity curves retain the same general shape for changes in the launch date. The initial periods of continuous sunlight simply slide along the launch right ascension axis.

In figure 16 is shown a cross plot of the information given in figures 12 through 15 for several selected launch right ascensions. As can be seen, in all cases the maximum eccentricity varies nearly sinusoidally with respect to the launch date. This allows a relatively accurate interpolation of this information for launch dates between the actually computed dates. The sinusoid for the 320-degree launch right ascension is seen to be
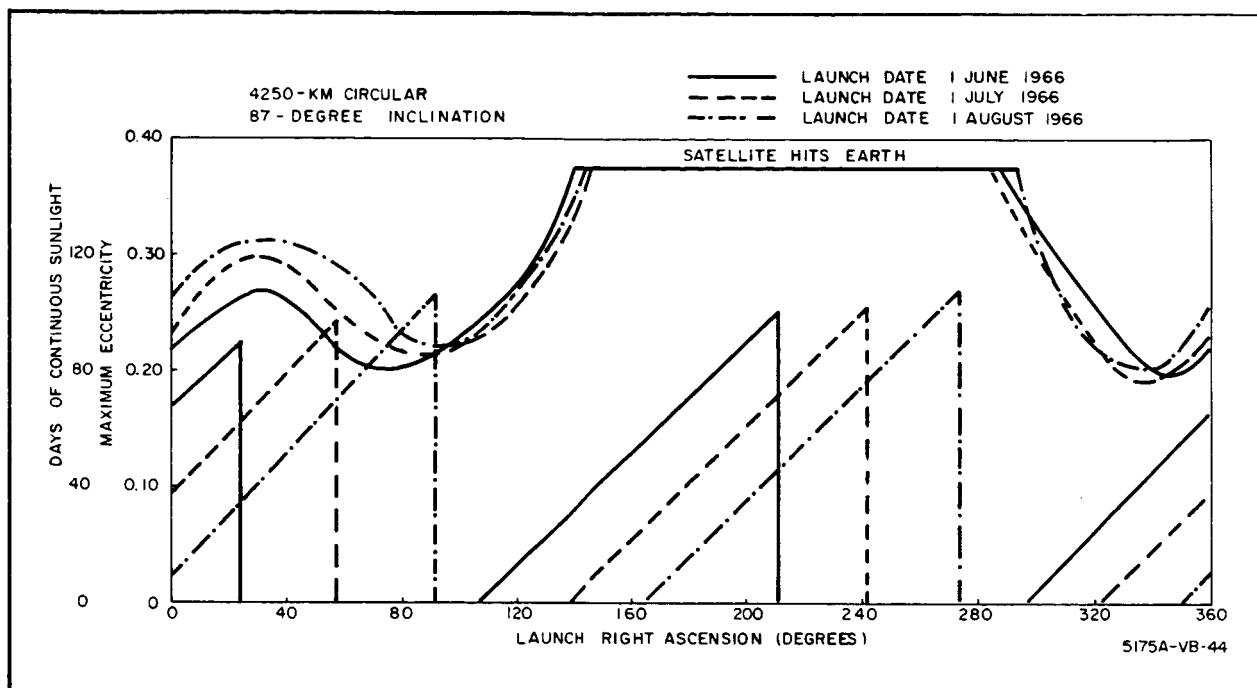
Figure 12.   Maximum Eccentricity vs Launch Right Ascension -
June, July, August
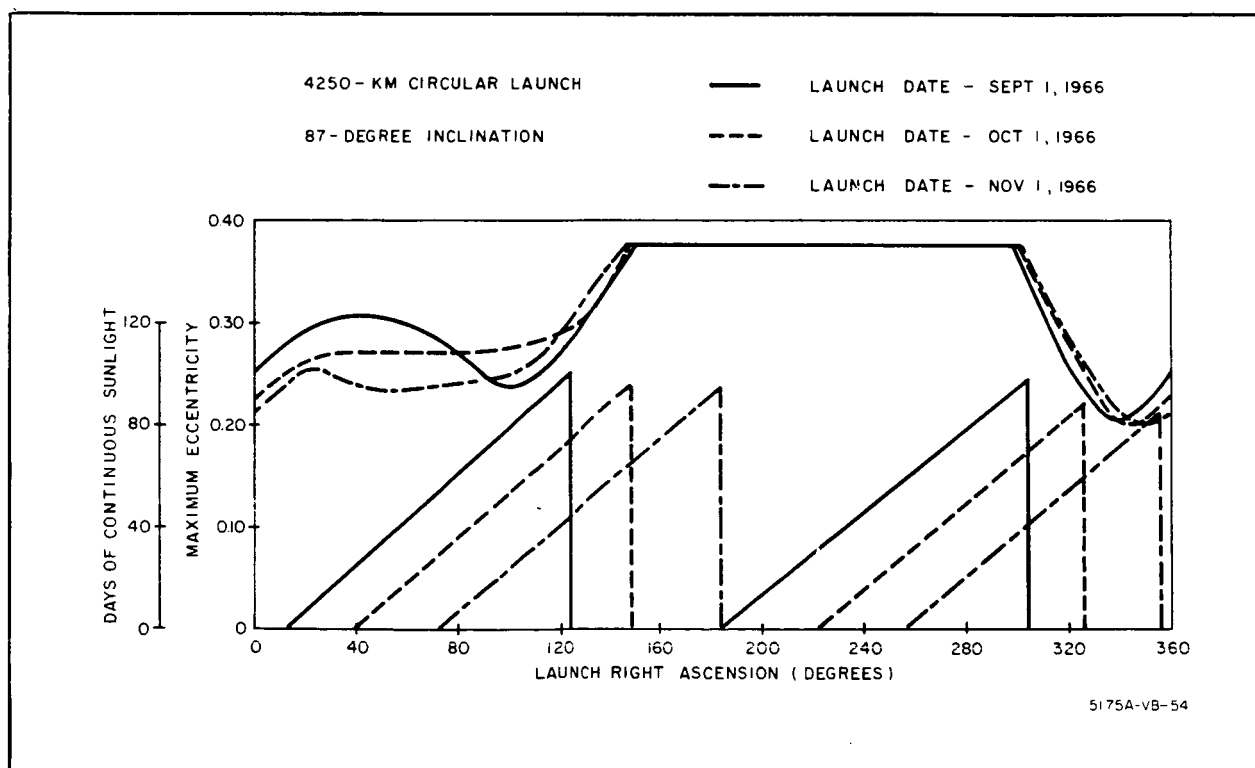
Figure 13.   Maximum Eccentricity vs Launch Right Ascension -
September, October, November

33
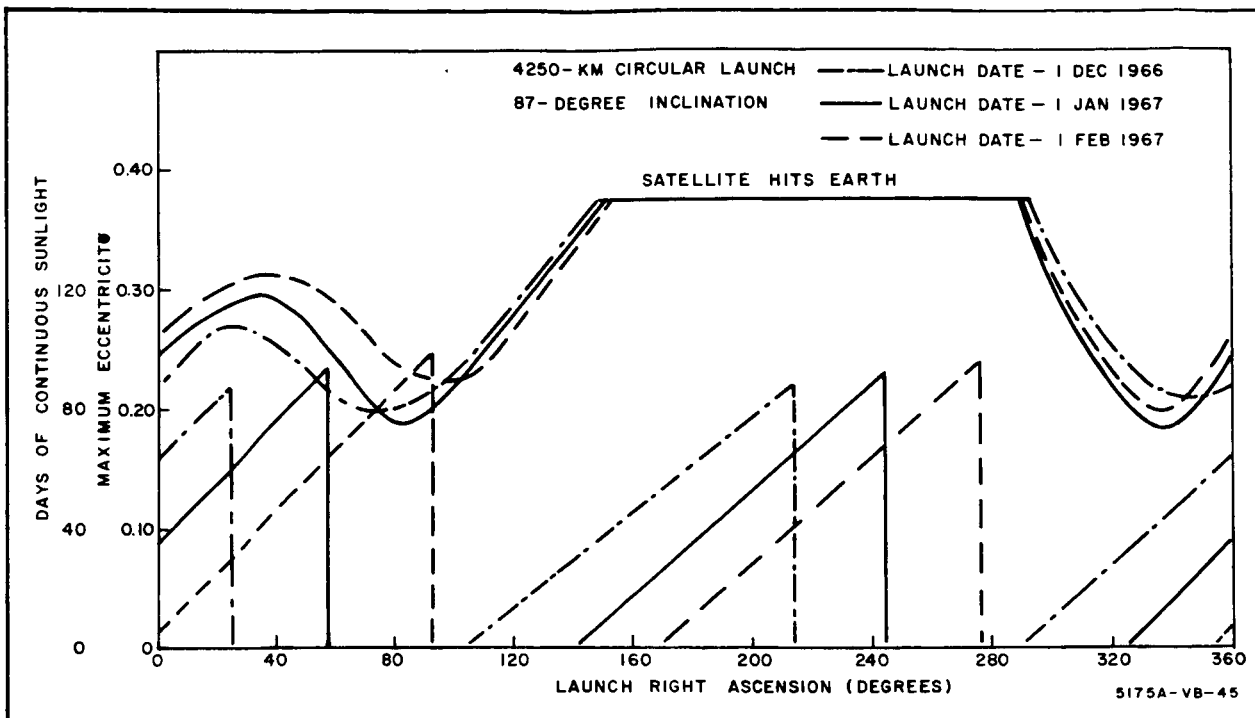
Figure 14. Maximim Eccentricity vs Launch Right Ascension -
December, January, February
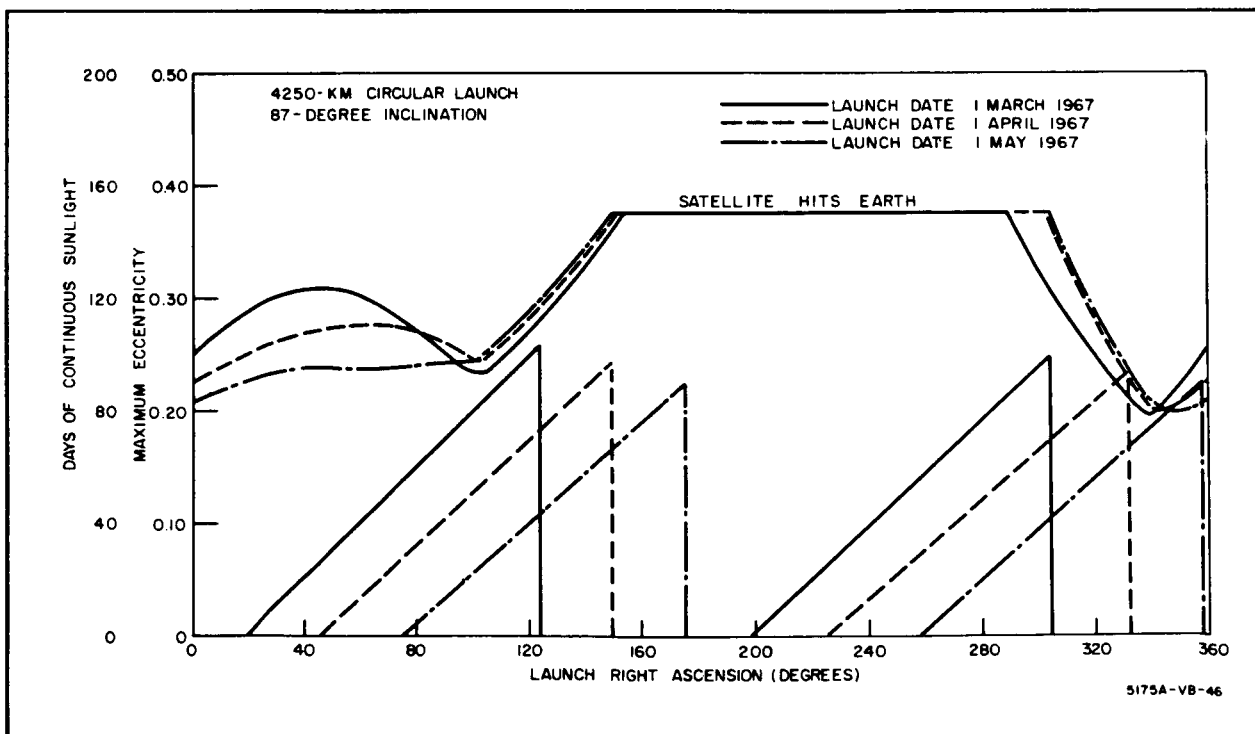
Figure 15. Maximum Eccentricity vs Launch Right Ascension -
March, April, May

34
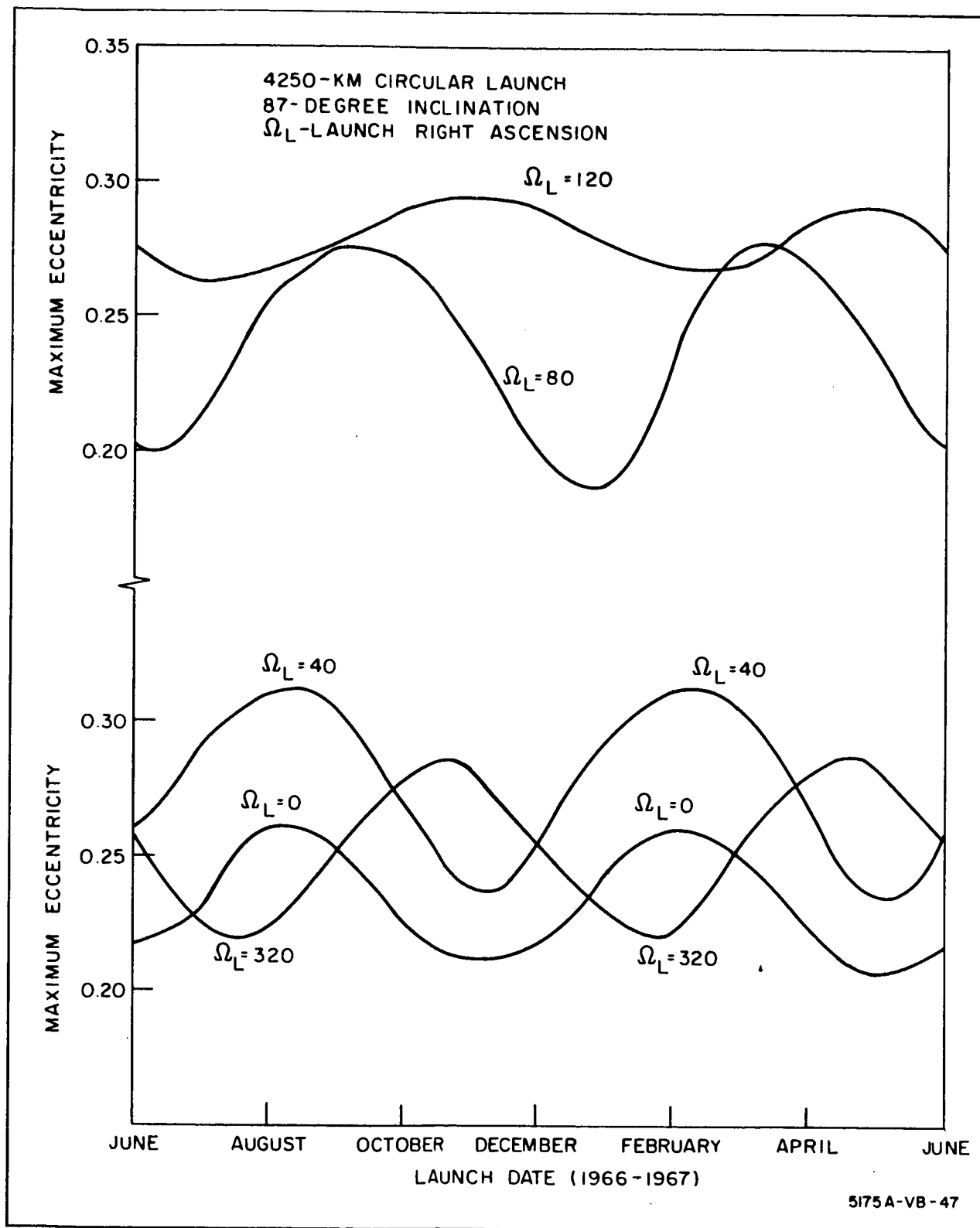
Figure 16. Correlation of Maximum Eccentricity to Launch Date

approximately 180 degrees out of phase with respect to the other launch right ascensions. At approximately a launch right ascension of 340 degrees, the curve appears to flatten out and shift its phase relationship. The flattening of the 120-degree curve indicates the approaching of another transition point where the phase relationship will be shifted again.

Figure 17 shows the nominal launch right ascension which should be chosen as a function of the time of year, and the maximum eccentricity associated with that orbit. These right ascensions were chosen from the two considerations of minimum eccentricity and fulfillment of the 14-day continuous sunlight requirement.

Generally, there are two regions of launch right ascension around 340 and 90 degrees where the maximum eccentricity reaches a minimum. In all cases, one or the other of the two regions of continuous sunlight occurs near one of the minimum eccentricity points. Figure 17 was generated,
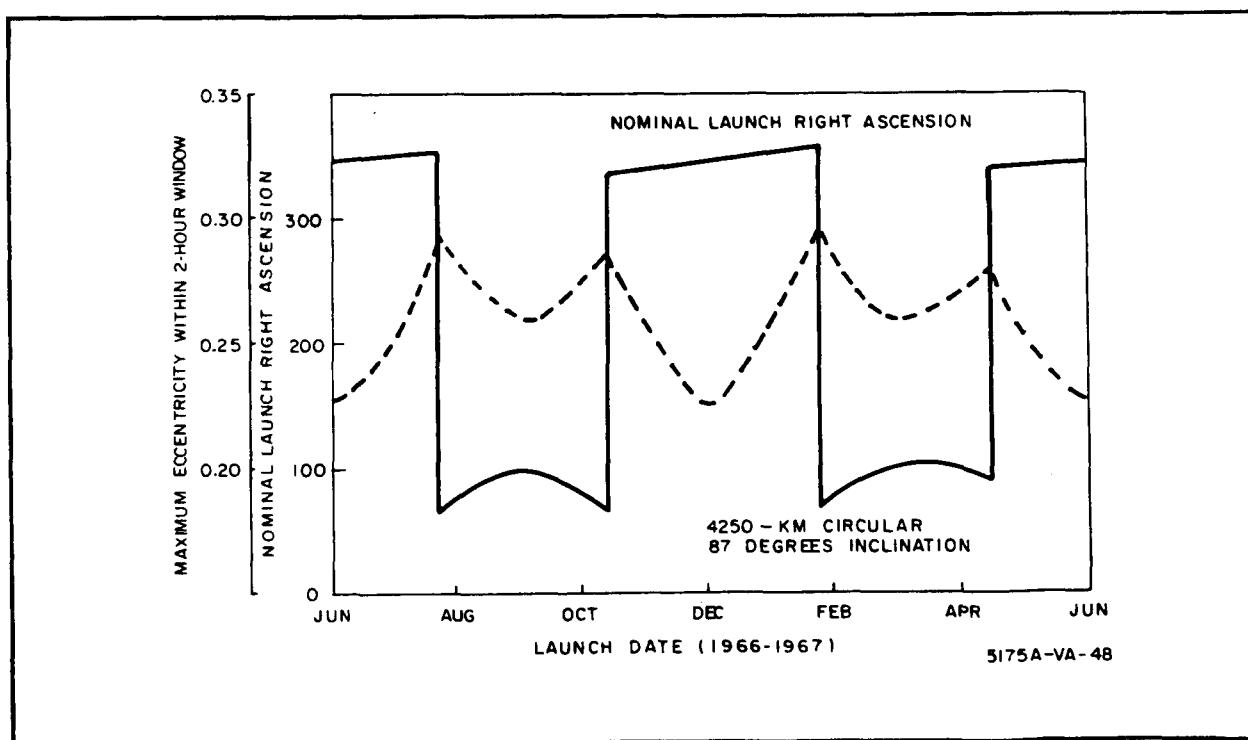
Figure 17. Launch Right Ascension vs Launch Date

then, by determining the right ascension which produces the least eccentricity within a 2-hour launch window (30-degree right ascension) and fulfills the continuous sunlight requirement. The points of discontinuity on the curve are points of transition from one sunlight region to the other.

As can be seen, there will be a penalty in terms of eccentricity for launching during certain times of the year. The eccentricity within the 2-hour launch window is seen to vary between about 0.225 and 0.295. The perigee altitude can be calculated by the expression:

$$p = a(1-e) - R_e$$

where a is the semimajor axis of the orbit and R_e is the radius of the earth. For the 4250-kilometer orbit the perigee altitude will be equal to 2225 kilometers and 1060 kilometers for eccentricities of 0.20 and 0.30, respectively. Considering then, that the Echo I satellite reaches a perigee altitude of approximately 1000 kilometers, even the maximum eccentricity of 0.295 is not considered to seriously affect the lifetime of the satellite. The previously shown results of the 7-station time sampled run for a 1 October launch date indicates that this added eccentricity will not seriously degrade the number of available observations.

In figure 18 are shown the effects of other launch errors on the maximum eccentricity of the orbit. Four different arguments of perigee were examined for a launch eccentricity of 0.02 (over twice the 3-sigma limit). As can be seen, the initial eccentricity is either added or subtracted from the nominal eccentricity depending upon the initial orientation of the perigee. In the case of the altitude error, an error on the low side by 100 kilometers is seen to increase the maximum eccentricity by approximately 0.03, while a 100-kilometer error on the high side does not affect the eccentricity appreciably. For the inclination error, the maximum eccentricity is seen to increase by nearly 0.02 per degree of inclination between 86- and 88-degrees
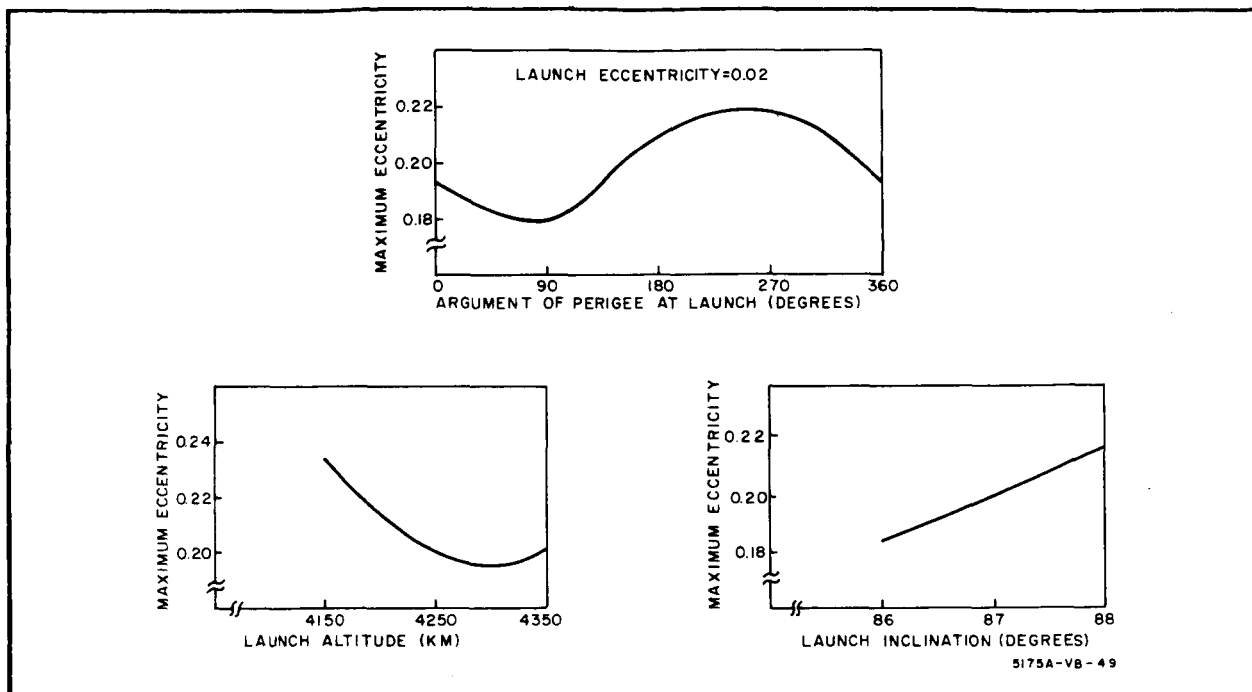
Figure 18. Launch Error Effects on Maximum Eccentricity

inclination. In all cases, the errors studied appear to be well outside the 3-sigma limit on the launch conditions and even for these extreme errors the eccentricity does not increase enough to degrade either the lifetime of the satellite or the number of available observations.

In figures 19 through 22 are shown the actual variations of the orbital elements as a function of time for an orbit with the nominal launch conditions. As can be seen, the altitude varies a total of 60 kilometers and the inclination a total of 3 degrees over the 5-year period of interest.

Validity of Sampling

Several tables were prepared to check the validity of the sampling techniques employed in the abbreviated runs. Tables VI and VII present the results of investigating the time sampling procedure. Considering only the four interior baselines and the five triangles of the 7-station problem, results obtained from the sampled run using orbit No. 1 compared with those
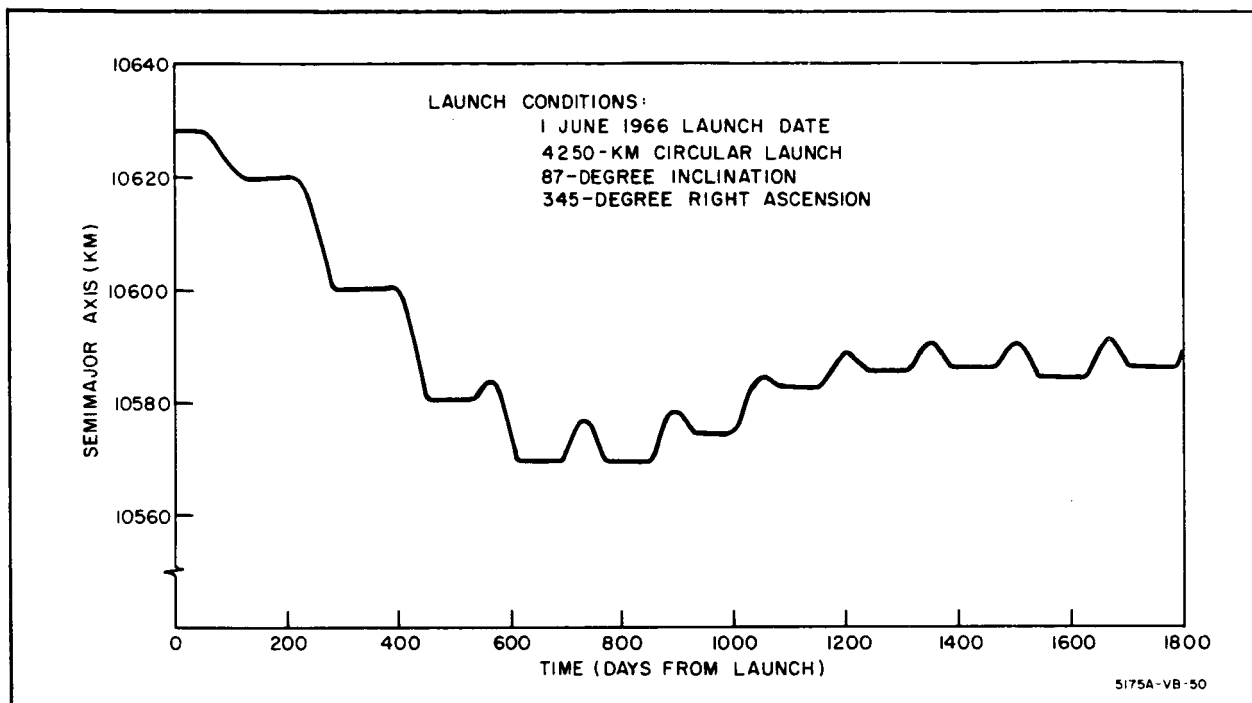
Figure 19. Semimajor Axis of Nominal Orbit as a Function of Time
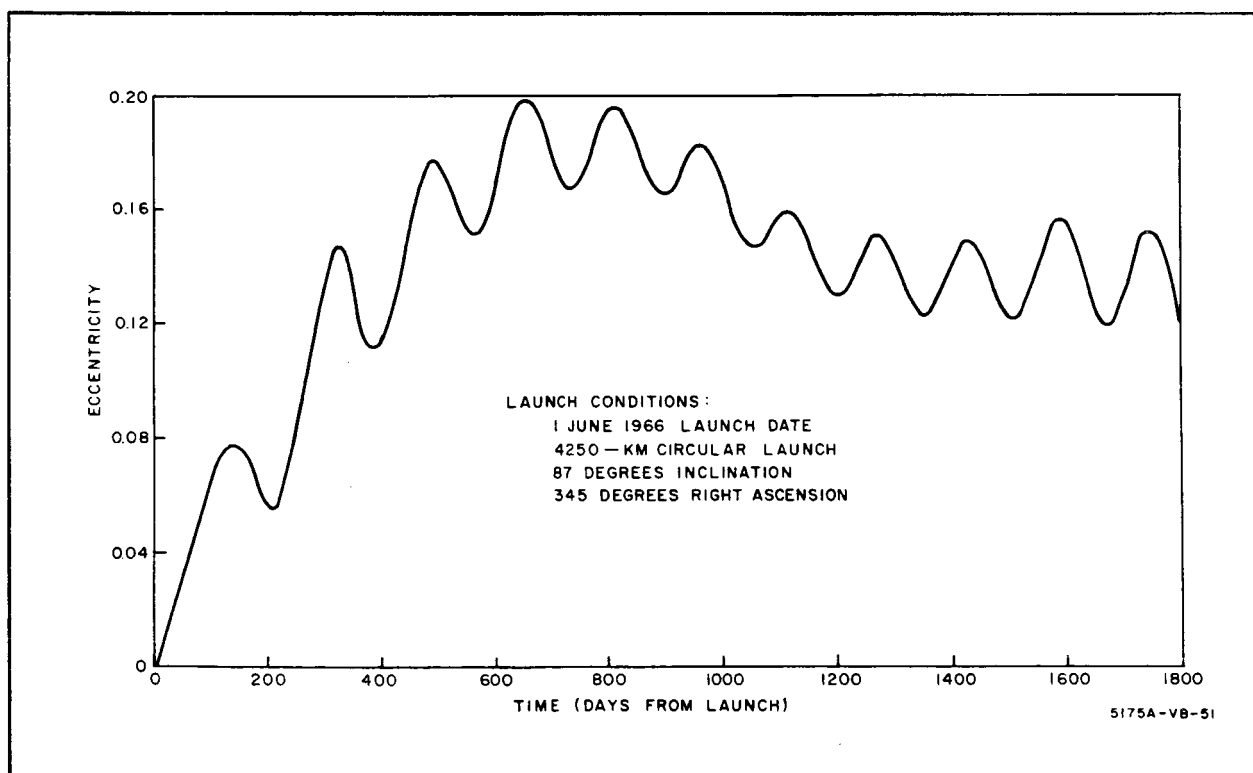
Figure 20. Eccentricity of Nominal Orbit as a Function of Time

LAUNCH CONDITIONS :
   I JUNE 1966 LAUNCH DATE
   4250 — KM CIRCULAR LAUNCH
   87 — DEGREES INCLINATION
   345 — DEGREES RIGHT ASCENSION

5I75A-VB-52
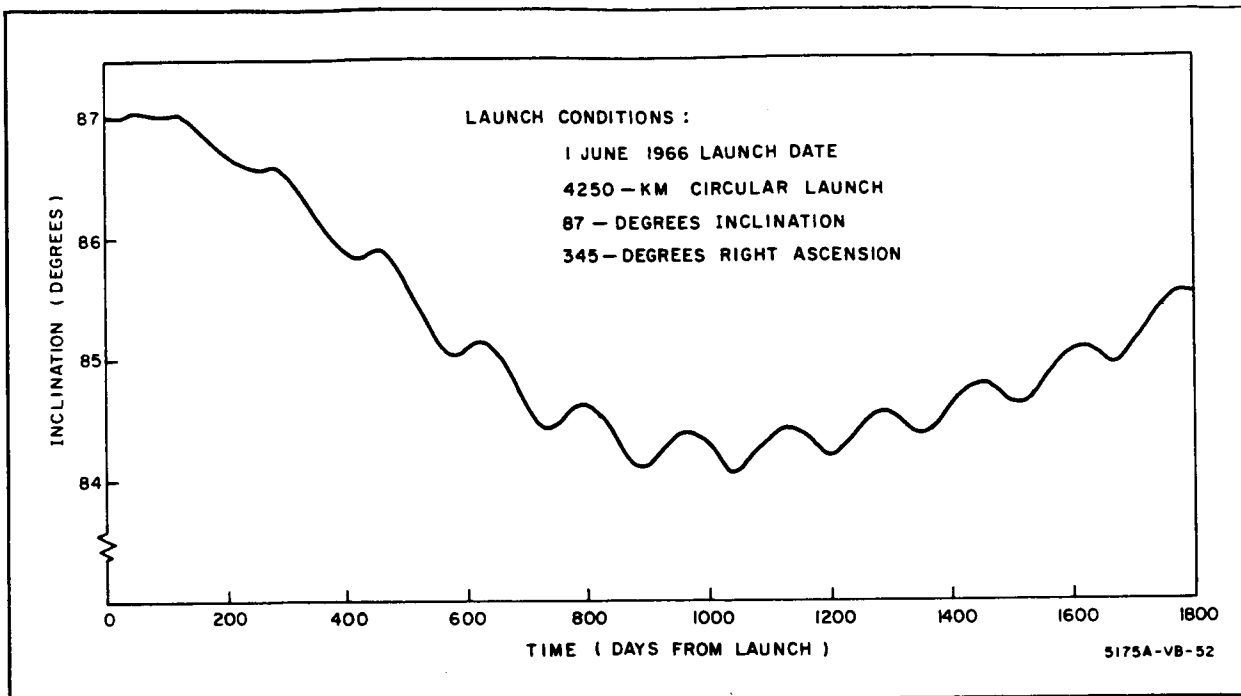
Figure 21.   Inclination of Nominal Orbit as a Function of Time

LAUNCH CONDITIONS: I JUNE 1966 LAUNCH DATE   87-DEGREE INCLINATION
   4250 KM CIRCULAR LAUNCH   345-DEGREE RIGHT ASCENSION

RIGHT ASCENSION

ARGUMENT OF PERIGEE

5I75A-VB-53
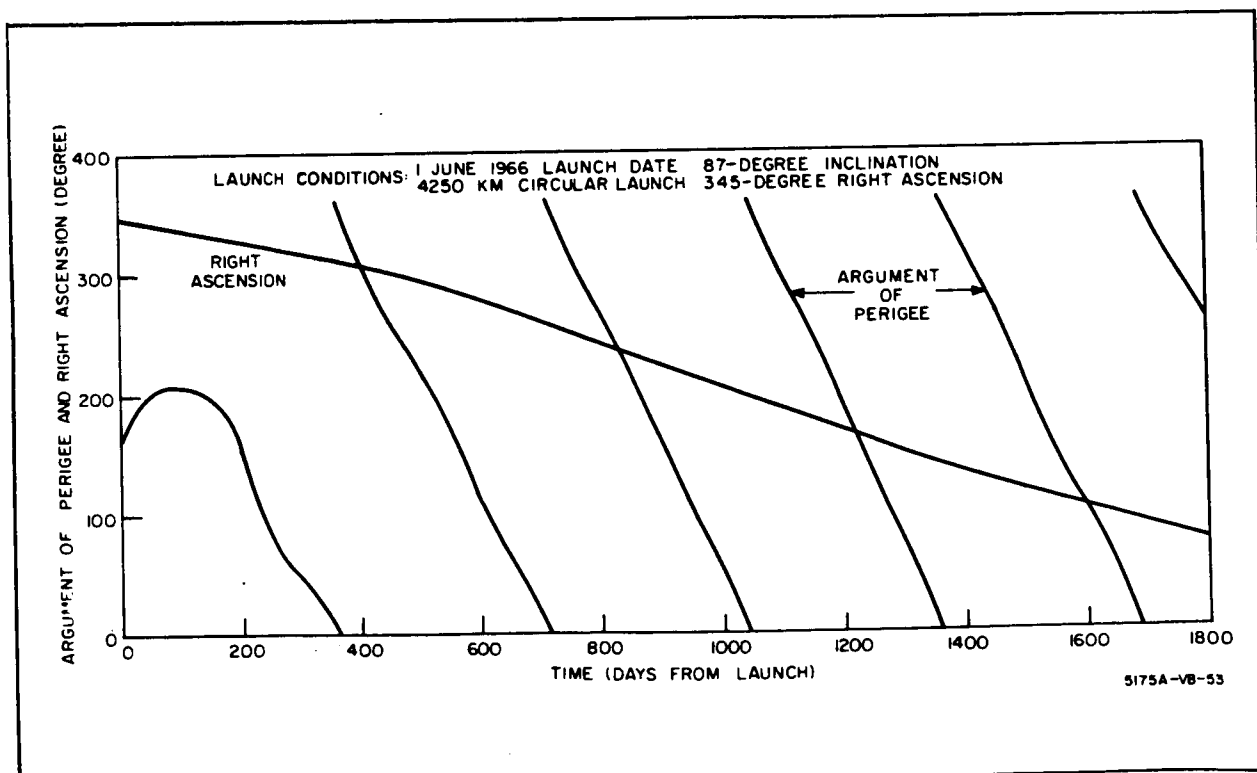
Figure 22.   Argument of Perigee and Right Ascension of Nominal Orbit
as a Function of Time

TABLE VI

BASELINE RESULTS

Baselines		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965															
		610	700	790	880	970	1060	1150	1240	1330	1420	1510	Total	1/3 Total	1st Year	2nd Year	3rd Year
17-18 (2-3)	G	6		24	3	15	21				33	33	135		33	36	66
	M	3		12	3	21		6			9		54		18	27	9
	+	6		28		6					8		48		34	6	8
S	-				6		15					15	36		6	15	15
17-18 (2-3)	G	6	12	25	4	6	20	10	7	19		2	111		47	43	21
	M	1		7	1		2	5	3	1	1	7	28		9	10	9
	+	2		10	1	1	1			3		4	22		13	2	7
T	-		9	2	5	1	10	4	4	12			47		16	19	12
18-27 (3-4)	G				48			6				6	60		48	6	6
	M		3										3		3		
	+				12								12		12		
S	-				48								48		48		
18-27 (3-4)	G		2	4	15			6					27		21	6	6
	M	2									2		4		2		2
	+				3								3		3		
T	-		1	3	14			4					22		18	4	
27-28 (4-5)	G	15	3		36								54		54		
	M	6	3										9		9		
	+	9											9		9		
S	-				27								27		27		
27-28 (4-5)	G	6		2	15	6	1		12				42		23	19	
	M	2		1	5				10	6	13		37		8	10	19
	+	4			1				1				6		5	1	
T	-			2	9								11		11		
28-35 (5-7)	G	108	27			66				75			276		135	66	75
	M					4							4			4	
	+																
S	-	177	33			132				69			411		210	132	69
28-35 (5-7)	G	36	7		48	42	7		50	44			234		91	99	44
	M				1	1							2		1	1	
	+	1							1	6			11		1	4	6
T	-	66	8		80	45	7		70	41			317		154	122	41

TOTAL GOOD OBSERVATIONS   SAMPLED RUN 525   270   108   147

                         TOTAL RUN   414   182   167   65

TABLE VII

TRIANGLE RESULTS

Triangles and Baselines		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965												1/3 Total	1st Year	2nd Year	3rd Year
		610	700	790	880	970	1060	1150	1240	1330	1420	1510	Total	1/3 Total	1st Year	2nd Year	3rd Year
8-17-18 (1-2-3)	G				3		21						24		3	21	
	M					3	3					3	9			6	3
	+				3		18						21		3	18	
	−						9						9			9	
	+				3		21						24		3	21	
	−																
	+																
	− (S)				6		27						33		6	27	
8-17-18 (1-2-3)	G	6	3	1	1			5					16		11	5	
	M	2			1	1	1			4			9		3	2	4
	+	3	2		1								6		6		
	−	2	2	1				3					8		5	3	
	+	8	3	1	1			3					16		13	3	
	−																
	+																
	− (T)	8	3	1	1			7					20		13	7	
18-27 -28 (3-4-5)	G		15										15		15		
	M		9										9		9		
	+																
	−		15										15		15		
	+		12										12		12		
	−																
	+																
	− (S)		9										9		9		
18-27 -28 (3-4-5)	G	6	7				1						14		13	1	
	M	2	2			2	5						11		4	7	
	+	2											2		2		
	−	5	6				1						12		11	1	
	+	8	3				2						13		11	2	
	−																
	+																
	− (T)	4	8				1						13		12	1	
28-29 -35 (5-6-7)	G	117				102				9			228		117	102	9
	M	3											3		3		
	+	192				189				9			390		192	189	9
	−	3											3		3		
	+	9				24							33		9	24	
	−	174				105			12				291		174	105	12
	+					12				3			15			12	3
	− (S)																
28-29 -35 (5-6-7)	G	82			27	65		9		10			193		109	74	10
	M	2				5							7		2	5	
	+	29				12							41		29	12	
	−	101			37	74		9		11			232		138	83	11
	+	142			38	113		18		14			325		180	131	14
	−	1											1		1		
	+	9				9							18		9	9	
	− (T)	3								4			7		3		4

TOTAL GOOD OBSERVATIONS

	Total	1st Year	2nd Year	3rd Year
SAMPLED RUN	267	133	123	9
TOTAL RUN	223	135	80	10

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obtained from the first 3 years of the full 5-year run (through interval 1510). In these tables, the results of the sampled runs were multiplied by 3, since the sampling interval was 1 month in 3, and were listed in the rows marked S. The corresponding results from the complete run were listed below in the rows marked T, the results being summed over the appropriate 3-month intervals. A fair degree of correspondence may be noted on comparing the total numbers of good observations obtained, although the correlations are not always high in the individual intervals. Several reasons for this were suggested by examination of the results and the computer output data. First, in the single-station observation program, a ficticious satellite position is introduced, since Lifetime-18 does not integrate satellite position. These ficticious positions tended to drift apart in the two runs (the sampled run and the complete run), and as a result of this drift the single station and multiple station observation tables were no longer in agreement on a month-to-month basis. This difference should more or less average out over long time periods, and is of questionable significance since the ficticious satellite positions have no real physical meaning at all. Secondly, the sampled intervals always occurred in June, September, December, and March, and thus the large numbers of observations obtained near the poles during the solstices, when multiplied by the factor three, tended to bias the sampled results high as compared with the results of the complete run. Since this factor would affect all sampled runs approximately equally, its effect on the comparison of the various orbits is diminished.

Table VIII lists results applicable to the investigation of the validity of the 7-station sample. The original 7-station group was labeled group A, while two more 7-station groups were chosen for comparison. These were stations 1, 3, 4, 10, 11, 20, and 21 (group B) and stations 9, 10, 19, 20, 29, 30, 36 (group C). Using the results of the complete 5-year run, the total number of good baseline observations through interval 1510 were calculated for these three groups and for the complete 36-station network.

TABLE VIII

7-STATION SAMPLE RESULTS

	Total Good Observations**	Average Good Observations ** Per Baseline
7 Station Samples*		
Group A	1078	99
Group B	932	85
Group C	789	72
All Stations*	8934	88

*Data obtained from 5-year run
**Through time interval 1510

Corresponding averages were found by dividing by the number of baselines. Again, fair correspondence may be noted, although not as good as might be wished.

From the above comparisons it may be concluded that although the sampling may have exaggerated the variations between orbits, the comparison of orbits on the basis of the abbreviated problem is probably valid. However, the possibility is not eliminated that a comparison on the basis of complete runs might have led to a somewhat different ranking of the top few orbits.

CONCLUSIONS

Six promising orbits were investigated in terms of the relative number of viewing opportunities provided to a representative 7-station network, using a time sample of 1 month in 3. The best of these orbits was then selected as a final orbit; this orbit has the following characteristics at launch:

Altitude - 4250 km (circular)

Inclination - 87 degrees

Right Ascension of the Ascending Node - 345 degrees

Launch Date - 1 June 1966 (Launch time approximately 6:30 AM Pacific Standard Time if launched from Pacific Missile Range)

It was noted during this selection that all orbits appeared to give satisfactory viewing opportunities, with only small variations from orbit to orbit.

A complete 5-year run using the selected orbit showed that it meets all observation requirements, and that the results of the short run agree reasonably well with those of the complete run. An investigation of variations in these characteristics, in terms of the resulting maximum eccentricity attained by the orbit during a 5-year period, revealed only slight changes for initial eccentricities of 0.02, for altitude variations of ± 100 km, and for inclination variations of ± 1 degree.

An additional facet of the study was an investigation of the maximum eccentricity attained during a 5-year period as a function of launch right ascension and launch date. Choosing the launch right ascension as a function of launch date such that this maximum eccentricity is made as small as possible, the resulting value of maximum eccentricity varies with launch date, and is smallest for June and December launches.

APPENDIX I

GROUND STATION, SATELLITE, SUNLINE ANGLE TEST

The angle that the line of sight of the camera makes with the sunline vector can be critical to the success of the observation. In particular, if this angle exceeds 135 degrees, the image of the sun on the satellite becomes so distorted that it reduces the accuracy of the data reduction techniques. Consequently, for successful observations this angle must be less than 135 degrees.

The geometry of the situation is shown in figure 23. In this figure the satellite is shown to be in the plane of the station vector, $\overline{r}_i$, and the sunline vector, $\overline{r}_s$. The radius vector to the satellite is defined by $\overline{r}$ and the camera line of sight vector, $\overline{S}$ is simply:

$$\overline{S} = \overline{r}_i - \overline{r}$$

Then the station, satellite, sun angle, γ, is defined by:

$$\cos \gamma = \frac{\overline{r}_s \cdot \overline{S}}{r_s S}$$

Consider now a procedure for determining the situation under which the angle γ reaches a maximum. Let a coordinate system be defined such that the x axis lies along the station vector, $\overline{r}_i$, and the y axis is perpendicular in the plane of the paper (station-sunline vectors). In this coordinate system the station, satellite, and sunline vectors can be expressed as follows:

$$\overline{r}_i = \begin{bmatrix} R_i \\ 0 \\ 0 \end{bmatrix} \qquad \overline{r} = \begin{bmatrix} r \cos \theta_r \\ r \sin \theta_r \\ 0 \end{bmatrix} \qquad \overline{r}_s = \begin{bmatrix} r_s \cos \theta_s \\ r_s \sin \theta_s \\ 0 \end{bmatrix}$$

The situation where the satellite is not in the plane of the paper can be considered by rotating the radius vector, $\overline{r}$, about the z axis through an arbitrary angle θ_z. This yields for the radius vector and the camera line of sight vector:
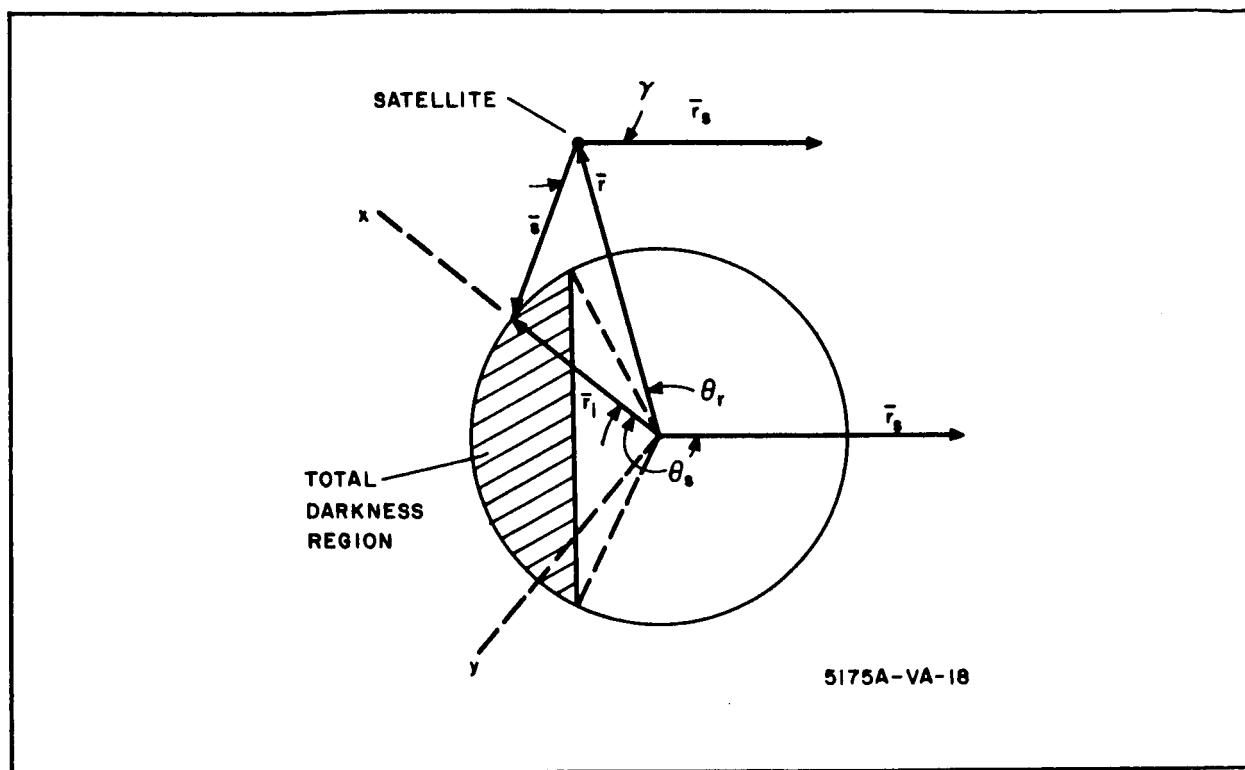
Figure 23. Ground Station, Satellite, Sunline Angle Geometry

$$\bar{r} = \begin{bmatrix} r \cos \theta_r \\ r \sin \theta_r \cos \theta_z \\ r \sin \theta_r \sin \theta_z \end{bmatrix} \qquad \bar{S} = \begin{bmatrix} R_i - r \cos \theta_r \\ -r \sin \theta_r \cos \theta_z \\ -r \sin \theta_r \sin \theta_z \end{bmatrix}$$

and the station, satellite, sunline angle is:

$$\cos \gamma = \frac{\cos \theta_s (R_i - r \cos \theta_r) - r \sin \theta_s \sin \theta_r \cos \theta_z}{\left(R_i^2 - 2 R_i r \cos \theta_r + r^2\right)^{1/2}}$$

The rate of change of this angle with respect to the out-of-plane angle, θ_z, is:

$$- \sin \gamma \frac{d\gamma}{d\theta_z} = \frac{r \sin \theta_s \sin \theta_r \sin \theta_z}{\left(R_i^2 - 2 R_i r \cos \theta_r + r^2\right)^{1/2}}$$

Thus, the point of maximum γ with respect to the angle θ_z is independent of the other variables and occurs at the point where θ_z is zero, or in the plane of the station, sunline vectors.

I-2

The geometry of figure 23 is simplified to show the conditions for maximum angle in figure 24. Maximum angle will occur when the station vector, $\overline{r_i}$, is on the 18-degree shadow terminator making the angle θ_s equal to 108 degrees. The elevation angle, ϵ, must also be a minimum for the station, satellite, sun (SSS) angle to be a maximum. From the geometry of the dashed triangle, if the SSS angle is 135 degrees, the elevation angle must be 27 degrees. Furthermore, if the SSS angle increases, the elevation angle must decrease.
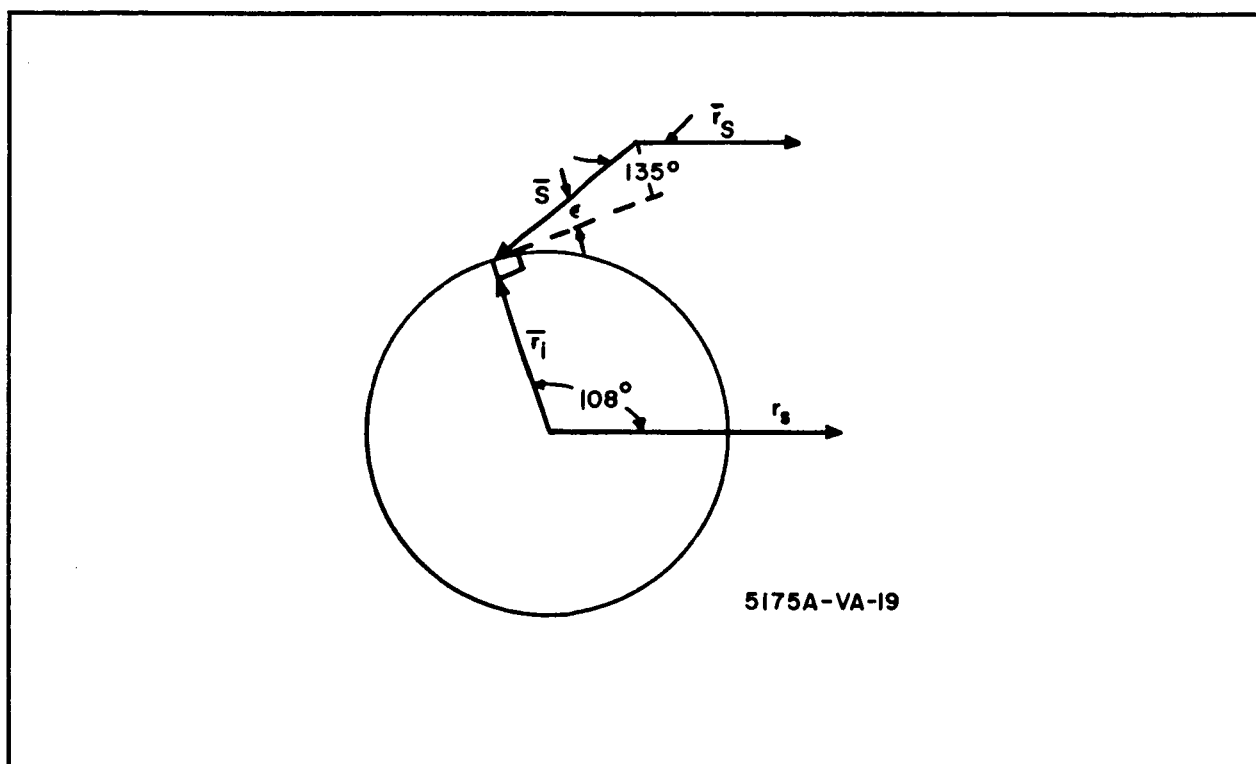
Figure 24. Geometry at Maximum Angle Point

From this discussion it can be concluded that the station, satellite, sun angle can be greater than 135 degrees only for an elevation angle less than 27 degrees. Since this affects only 2 degrees of the marginal observation, and since the choice of orbits is not predicated on the marginal observations, this condition has been ignored in the computer programs.

APPENDIX II

CALCULATION OF SUNLINE VECTOR

The position of the sun relative to the earth must be known to calculate the station darkness times. A reasonably accurate position can be obtained through a Keplerian evaluation of the position of the earth in its orbit about the sun at the specified time. The orbital elements of the earth are:[*]

$$n_E = 0.9856091 \text{ deg/day (mean angular motion)}$$

$$e_E = 0.01672592 \text{ (eccentricity)}$$

$$\omega_E = 102.26498 \text{ degrees (argument of perigee)}$$

$$T_{O_E} = \text{January } 2.62733, \ 1960 \text{ (epoch)}$$

Then at any time, t, the position of the earth in heliocentric coordinates can be determined by the following Keplerian transformations:

$$M_E = n_E (t - T_{O_E})$$

$$E_E = M_E + e_E \sin E_E$$

$$u_E = \tan^{-1} \left[\frac{\sqrt{1 - e_E^2} \sin E_E}{\cos E_E - e_E} \right]$$

$$r_E = a_E (1 - e_E \cos E_E)$$

$$\overline{r}_E = \begin{bmatrix} r_E \cos (\omega_E + u_E) \\ r_E \sin (\omega_E + u_E) \\ 0 \end{bmatrix}$$

where M_E, E_E, and u_E are the mean, eccentric, and true anomaly of the earth respectively, and r_E and $\overline{r}_E$ are the magnitude and vector components of the heliocentric position respectively.

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[*] V. M. Blanco and S. W. McCuskey, "Basic Physics of the Solar System," Addison-Wesley Publishing Company, Inc., p. 285, 1961.

The position of the sun in geocentric coordinates, $\bar{r}_s$, can be obtained by reversing the sign of the heliocentric position vector and correcting for the tilt of the earth's axis by rotating the vector around the x axis through the angle which the ecliptic plane makes with the plane of the equator.

$$r_s = - \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 23°\ 27'\ 8'' & -\sin 23°\ 27'\ 8'' \\ 0 & \sin 23°\ 27'\ 8'' & -\cos 23°\ 27'\ 8'' \end{bmatrix} \cdot \begin{bmatrix} r_{Ex} \\ r_{Ey} \\ r_{Ez} \end{bmatrix}$$

APPENDIX III

CALCULATION OF STATION DARKNESS TIMES

To calculate the station darkness times, the sunline and station vectors must be related on a time basis (figure 25). From the American Ephemeris, the hour angle of the Greenwich meridian is 100.43735 degrees at January 1.0, 1965 universal time. Using an earth rotation rate of 360.98561 degrees per day, the hour angle of the Greenwich meridian, h_G, at Greenwich midnight on any day can be expressed as:

$$h_G = 100.43735 + 0.98561 \, \Delta T_G$$

where ΔT_G is the integer number of days between the time of interest and January 1.0, 1965 universal time.
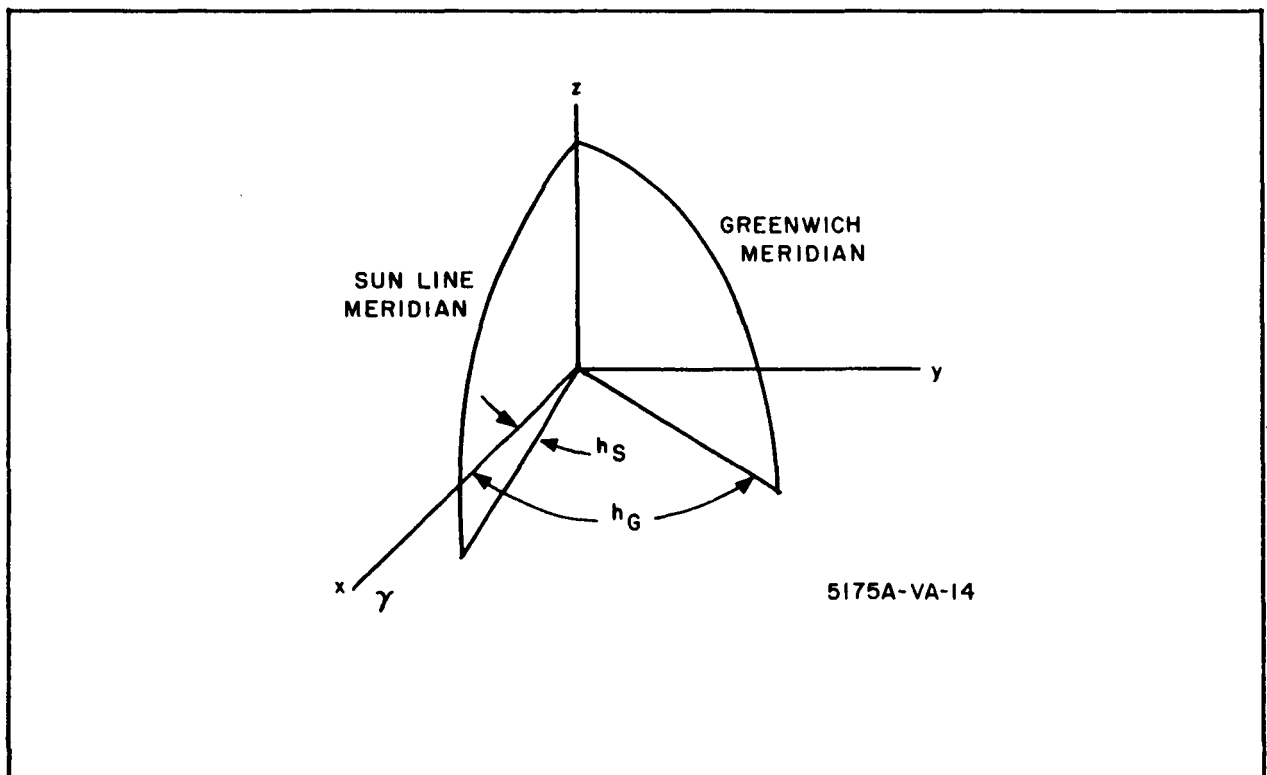
Figure 25. Relationship of Solar Vector to Greenwich Meridian

Assuming that the position of the sun is fixed over a day in the geocentric coordinate system and is defined by the vector, $\bar{r}_s$, (Appendix II), the hour angle of the sunline meridian is:

$$h_s = \tan^{-1}\left[\frac{r_{sy}}{r_{sx}}\right]$$

Then the longitude of the sun as a function of time throughout the day is:

$$\theta_s = h_s - h_G - 6.3003874\, t_h$$

where t_h is the Greenwich mean solar time in days. The latitude of the sun is assumed to remain constant over a day and is:

$$\phi_s = \tan^{-1}\left[\frac{r_{sz}}{\left(r_{sx}^2 + r_{sy}^2\right)^{1/2}}\right]$$

Both the sunline vector and the station vectors can be expressed as functions of their respective latitudes and longitudes in the same manner as shown in Appendix IV. Then the cosine of the angle between the two vectors, C_s, is simply the dot product of the two vectors or:

$$\cos C_s = \cos\phi_i \cos\phi_s \cos(\theta_s - \theta_i) + \sin\phi_i \sin\phi_s$$

By definition, when the angle C_s reaches 108 degrees, the station passes into the total darkness needed for a successful observation. Solving for the critical solar longitude which satisfies this condition yields:

$$\theta_s = \cos^{-1}\left[\frac{\cos 108° - \sin\phi_i \sin\phi_s}{\cos\phi_i \cos\phi_s}\right] + \theta_i$$

If the condition:

$$\frac{\cos 108° - \sin\phi_i \sin\phi_s}{\cos\phi_i \cos\phi_s} > 1$$

is met, no solution exists for the critical solar longitudes and the station is either completely in darkness or completely in sunlight for the entire day. The sunlight-darkness condition can be determined by evaluating the angle

C_s at any time of day. Choosing the time when the longitude of the sun is equal to the longitude of the station for simplicity yields the condition. If:

$$\sin \phi_i \sin \phi_s \le \cos 108°$$

then the station is always in darkness; otherwise the station is always in sunlight.

If the first condition is not met, the station will be in both darkness and sunlight during some portion of the day. Then the region of station darkness can be determined by calculating the times at which the critical solar longitudes occur. These times are determined by:

$$t_{d1} = - \frac{1}{6.3003874} \left[\cos^{-1} \left(\frac{\cos 108° - \sin \phi_i \sin \phi_s}{\cos \phi_i \cos \phi_s} \right) + (\theta_i + h_G - h_s) \right]$$

$$t_{d2} = - \frac{1}{6.3003874} \left[-\cos^{-1} \left(\frac{\cos 108° - \sin \phi_i \sin \phi_s}{\cos \phi_i \cos \phi_s} \right) + (\theta_i + h_G - h_s) \right]$$

and the status of the station between these time limits can be determined by checking the angle C_s at the midpoint. Thus if:

$$\cos \phi_i \cos \phi_s \cos \left[- \frac{t_{d1} + t_{d2}}{2(6.3003874)} + h_G - h_s \right] + \sin \phi_i \sin \phi_s \le \cos 108°$$

then the station is in total darkness between times t_{d1} and t_{d2}; otherwise the station is sunlit between these times.

APPENDIX IV

ENTRANCE AND EXIT OF SATELLITE FROM CONE OF OBSERVATION

The first step in solving for the entrance and exit times of the satellite into the cone of observation is to solve for the point of maximum elevation angle of the satellite with respect to the ground station. Then, if this angle is greater than the minimum limit, the two points of entrance and exit from the cone are found. In this manner the problem of trying to converge on a nonexistent solution is avoided when the maximum elevation angle is less than the required minimum.

To solve for the maximum elevation angle, the orbital elements, many related orbital quantities, and their derivatives are needed. At any time t the orbital elements of the satellite can be defined in the terrestrial coordinate system (Appendix XII) as follows:

$$a = a_L + (t - T_L) \, da/dt$$

$$e = e_L + (t - T_L) \, de/dt$$

$$\Omega_T = \Omega_{TL} + (t - T_L)(d\Omega/dt - 6.300696)$$

$$i = i_L + (t - T_L) \, di/dt$$

$$\omega = \omega_L + (t - T_L) \, d\omega/dt$$

where the orbital elements a_L, e_L, i_L, ω_L, and Ω_L are the orbital elements obtained from the "Lifetime 18" program at the reference time T_L. Their rates of change, da/dt, de/dt, etc, are defined as the average rate of change between two successive recordings of the orbital elements (1-day intervals). The terrestrial right ascension, Ω_{TL}, is defined as:

$$\Omega_{TL} = h_G + 6.3003874 (T_L - \Delta T_G) - \Omega_L$$

where h_G and ΔT_G are as defined in Appendix III. The mean angular motion, n, of the satellite is defined as:

$$n = \frac{K\sqrt{M}}{a^{3/2}}$$

where $K\sqrt{M}$ is the universal gravitational constant and M is the mass of the earth.

$$K\sqrt{M} = 631.3503 \ km^{3/2}/sec.$$

The eccentric anomaly to the point of nodal crossing, E_ω, can be defined through Keplerian mechanics to be:

$$E_\omega = \tan^{-1} \left[\frac{-\sin\omega\sqrt{1-e^2}}{e + \cos\omega} \right]$$

and the sine and cosine of this angle are defined as:

$$\sin E_\omega = \frac{-\sin\omega\sqrt{1-e^2}}{1 + e\cos\omega}$$

$$\cos E_\omega = \frac{e + \cos\omega}{1 + e\cos\omega}$$

Then the time of perigee passage, T_o, is:

$$T_o = T_n - \frac{E_\omega - e\sin E_\omega}{n}$$

where T_n is defined as the time of nodal crossing and is assumed to be fixed over an orbit; i.e., it does not vary as a function of the orbital perturbations within an orbit (derivative with respect to time is assumed to be zero). The nodal crossing time is updated from orbit to orbit in the manner described in the main body of the text. The rates of change of these quantities with respect to time are obtained by differentiating the respective quantities with respect to time and are:

$$\frac{dn}{dt} = -\frac{3}{2} \frac{K\sqrt{M}}{a^{5/2}} \frac{da}{dt}$$

$$\frac{de_\omega}{dt} = \frac{\sqrt{1-e^2}}{(1 + e\cos\omega)^2} \left[\frac{\sin\omega}{1-e^2} \left\{ 1 + e\cos\omega(1-e^2) \right\} \frac{de}{dt} - (1 + e\cos\omega) \frac{d\omega}{dt} \right]$$

$$\frac{dT_o}{dt} = \frac{n\sin E_\omega\, de/dt + (E_\omega - e\sin E_\omega)\, dn/dt - n(1 - e\cos E_\omega)\, dE_\omega/dt}{n^2}$$

The rate of change of the mean anomaly, dM/dt, is defined in a similar manner:

$$\frac{dM}{dt} = (t - T_o)\frac{dn}{dt} + n\left(1 - \frac{dT_o}{dt}\right)$$

and the mean anomaly, M, at any time is then defined as:

$$M = M_o + (t - T_n)\frac{dM}{dt}$$

where:

$$M_o = E_\omega - e \sin E_\omega$$

In this way an integrated effect is achieved on the mean anomaly rather than defining it in the standard manner as functions of the instantaneous mean angular motion and time of perigee passage.

The eccentric anomaly, E, of the satellite is defined by the equation:

$$E = M + e \sin E$$

This is, of course, Kepler's equation and must be solved iteratively to determine the eccentric anomaly.

The radius, r, to the satellite and the true anomaly, u, are defined as follows:

$$r = a(1 - e \cos E)$$

$$\sin u = \frac{\sqrt{1 - e^2} \sin E}{1 - e \cos E}$$

$$\cos u = \frac{\cos E - e}{1 - e \cos E}$$

Again the rates of change of these quantities are obtained by differentiating the above expressions which results in:

$$\frac{dE}{dt} = \frac{dM/dt + \sin E \, de/dt}{1 - e \cos E}$$

$$\frac{dr}{dt} = (1 - e \cos E)\frac{da}{dt} - a \cos E \frac{de}{dt} + ae \sin E \frac{dE}{dt}$$

$$\frac{du}{dt} = \frac{\sqrt{1 - e^2}}{1 - e \cos E}\frac{dE}{dt} + \frac{\sin E}{\sqrt{1 - e^2}}\frac{(1 - e - e^2)}{(1 - e \cos E)^2}\frac{de}{dt}$$

The latitude, ϕ, and longitude, θ, of the satellite as defined in Appendix XII are:

$$\phi = \tan^{-1}\left[\frac{\sin(u+\omega)\sin i}{\left\{\cos^2(u+\omega)+\sin^2(u+\omega)\cos^2 i\right\}^{1/2}}\right]$$

$$\theta = \tan^{-1}\left[\frac{\cos(u+\omega)\sin\Omega_T + \sin(u+\omega)\cos i \cos\Omega_T}{\cos(u+\omega)\cos\Omega_T - \sin(u+\omega)\cos i \sin\Omega_T}\right]$$

The rates of change of these angles can also be obtained by differentiation and are:

$$\frac{d\phi}{dt} = \frac{\cos(u+\omega)\sin i\left(\frac{du}{dt}+d\omega/dt\right)+\sin(u+\omega)\cos i\, di/dt}{\left\{1-\sin^2(u+\omega)\sin^2 i\right\}^{1/2}}$$

$$\frac{d\theta}{dt} = \frac{d\Omega_t}{dt} + \frac{\cos i\,(du/dt+d\omega/dt)-\sin(u+\omega)\cos(u+\omega)\sin i\, di/dt}{1-\sin^2(u+\omega)\sin^2 i}$$

The radius vector, r_i, from the center of the earth to the ith ground station can be expressed as:

$$\overline{r_i} = R_i\begin{bmatrix}\cos\phi_i\cos\theta_i\\\cos\phi_i\sin\theta_i\\\sin\phi_i\end{bmatrix}$$

where R_i is the distance from the center of the earth to the ground station, ϕ_i is the latitude, and θ_i is the longitude of the ground station. The radius vector, $\overline{r}$, to the satellite can be expressed similarly as:

$$\overline{r} = r\begin{bmatrix}\cos\phi\cos\theta\\\cos\phi\sin\theta\\\sin\phi\end{bmatrix}$$

Then the cosine of the angle, c, between the two vectors is the dot product or:

$$\cos c = \cos\phi_i\cos\phi\cos(\theta-\theta_i)+\sin\phi_i\sin\phi$$

and the rate of change of this angle is:

$$\sin c\,\frac{dc}{dt} = (\cos\phi_i\sin\phi\cos(\theta-\theta_i)-\sin\phi_i\cos\phi)\frac{d\phi}{dt}$$
$$+\cos\phi_i\cos\phi\sin(\theta-\theta_i)\frac{d\theta}{dt}$$
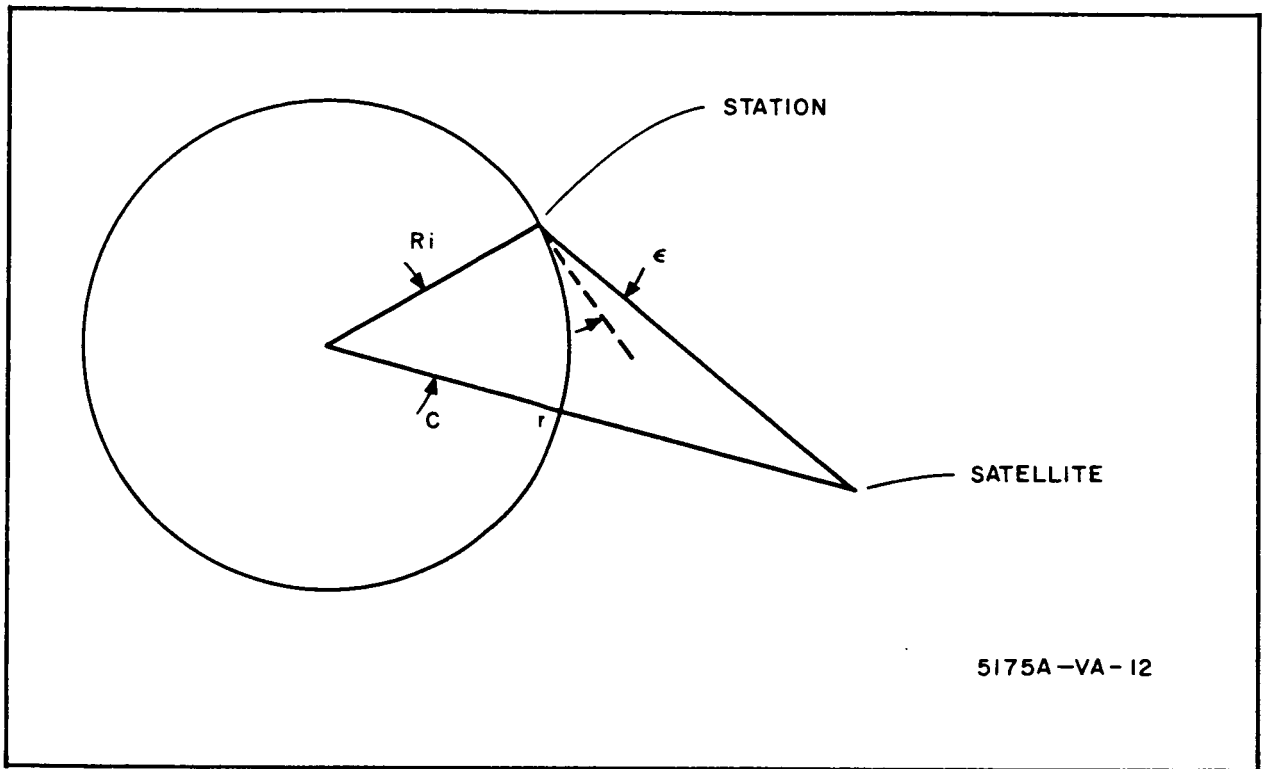
Figure 26. Geometry of Elevation Angle

Then the elevation angle, ϵ, can be determined as shown by the geometry of figure 26. A double application of the law of cosines to the triangle shown in the figure yields the elevation angle:

$$\cos (\pi/2 + \epsilon) = \frac{R_i - r \cos c}{(R_i^2 + r^2 - 2R_i r \cos c)^{1/2}} = - \sin \epsilon$$

By differentiating the expression for the elevation angle and equating the result to zero, an equation is obtained which yields the time at which the elevation angle is a maximum. Thus, the equation:

$$R_i r (1 - \cos^2 c) \frac{dr}{dt} + r^2 \sin c (R_i \cos c - r) \frac{dc}{dt} = 0$$

must be solved iteratively for time. If the elevation angle at this time is greater than the minimum limit, then the two times which satisfy the following equation will define the entrance and exit times of the satellite from the cone of observation.

$$\frac{R_i - r \cos c}{(R_i^2 + r^2 - R_i r \cos c)^{1/2}} = - \sin \epsilon_m$$

where ϵ_m is the minimum allowable elevation angle.

A modification of Newton's method is used to solve for the maximum elevation time. The entrance and exit times are solved through the use of a higher order iteration technique. It was found that by fitting a quadratic to two points and a slope in order to find the next trial point in the iteration process reduced the number of iterations required by approximately a factor of two, thus effecting a significant savings in machine time.

A method for obtaining an initial approximation to the maximum elevation time is described in Appendix VI. This approximation is used as a starter for the iteration process. The starters for the entrance and exit time iterations are chosen as an arbitrary point on either side of the maximum elevation time.

APPENDIX V

COURSE ELEVATION TEST

To eliminate the necessity of performing detailed computations on all stations, a test has been developed to detect those stations which cannot possibly see the satellite at an elevation angle greater than the minimum required. The test is developed from the geometry of figure 27, where $\overline{N}$ is a vector normal to the plane of the orbit and $\overline{r}_i$ is the station vector. The quantity h_a is defined as the maximum altitude the satellite can reach in an orbit and is:

$$h_a = a (1 + e)$$
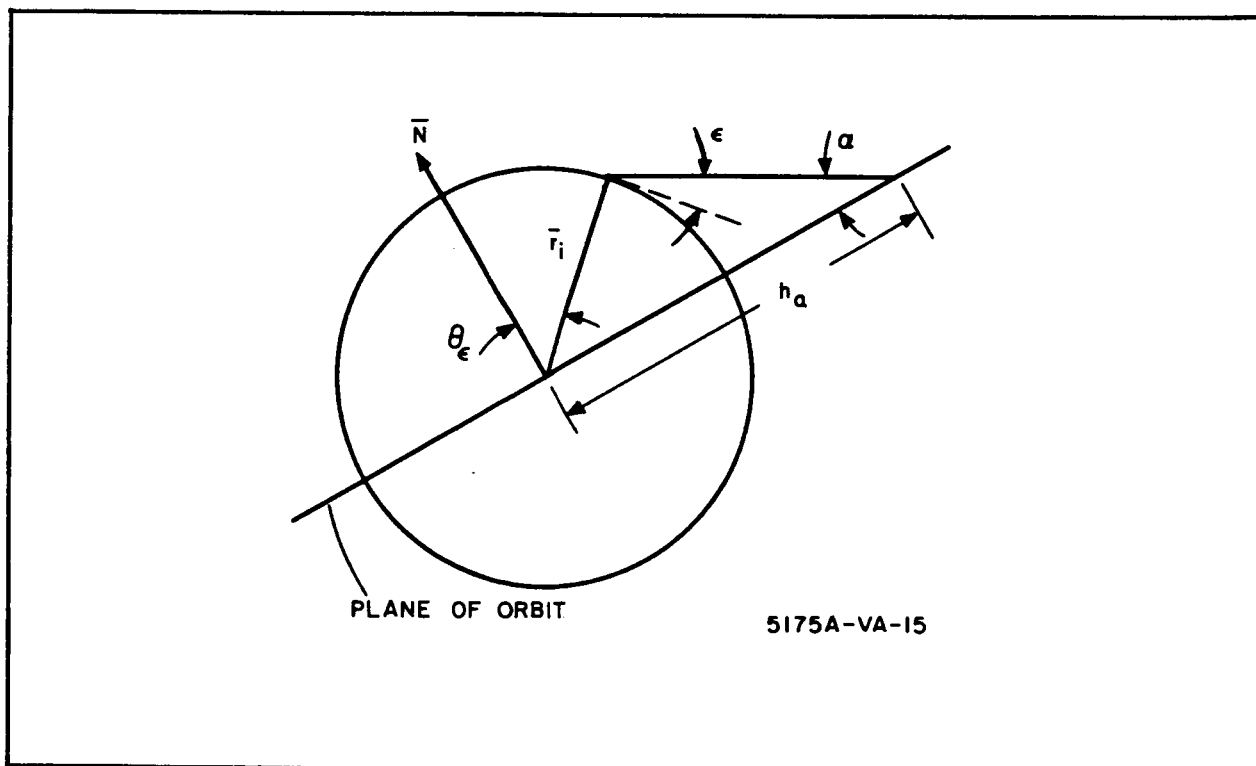
Figure 27. Geometry of Course Elevation Test

It can be seen that the maximum possible elevation angle from the station to the satellite would occur if the satellite were at its maximum altitude and

were in the plane of the station vector and normal vector. If the angle between the normal vector and the station vector is defined to be θ_ϵ, then the elevation angle, ϵ, under these conditions can be expressed as:

$$\tan \alpha = \frac{\cos \theta_\epsilon}{h_a / R_i - \sin \theta_\epsilon}$$

$$\epsilon = \theta_\epsilon - \alpha$$

where R_i is the magnitude of the station vector $\overline{r_i}$.

This maximum possible elevation angle has been evaluated for several different altitude ratios (h_a / R_i) as a function of the angle between the station vector and the normal vector (θ_ϵ). These results are shown in figure 28. As expected, the higher altitude ratios produce the higher maximum elevation angles.

Figure 29 shows the same data replotted to give the minimum allowable station-normal vector angle (θ_ϵ) needed for a possible observation under a 30-degree minimum elevation angle and a 25-degree minimum elevation angle (marginal observations).

In the program, the station-normal vector angle is evaluated at the estimated maximum elevation point (Appendix VI). Since there is some error in this estimation, a margin of safety must be added to the minimum allowable station-normal vector angle to ensure that at the actual maximum elevation time this angle will not have changed by enough to place it in the region of possible observation. The station-normal vector angle is defined by the dot product of the two vectors as:

$$\cos \theta = -\frac{\sin i \, (r_{ix} \sin \Omega_T - r_{iy} \cos \Omega_T + r_{iz} \cos i)}{R_i}$$

Then the rate of change of this angle can be expressed as (neglecting all derivatives except the rotation of the earth, $d\Omega_T / dt$):

$$\frac{d\theta_\epsilon}{dt} = \frac{\sin i \, (r_{ix} \cos \Omega_T + r_{iy} \sin \Omega_T)}{R_i \sin \theta_\epsilon} \frac{d\Omega_T}{dt}$$
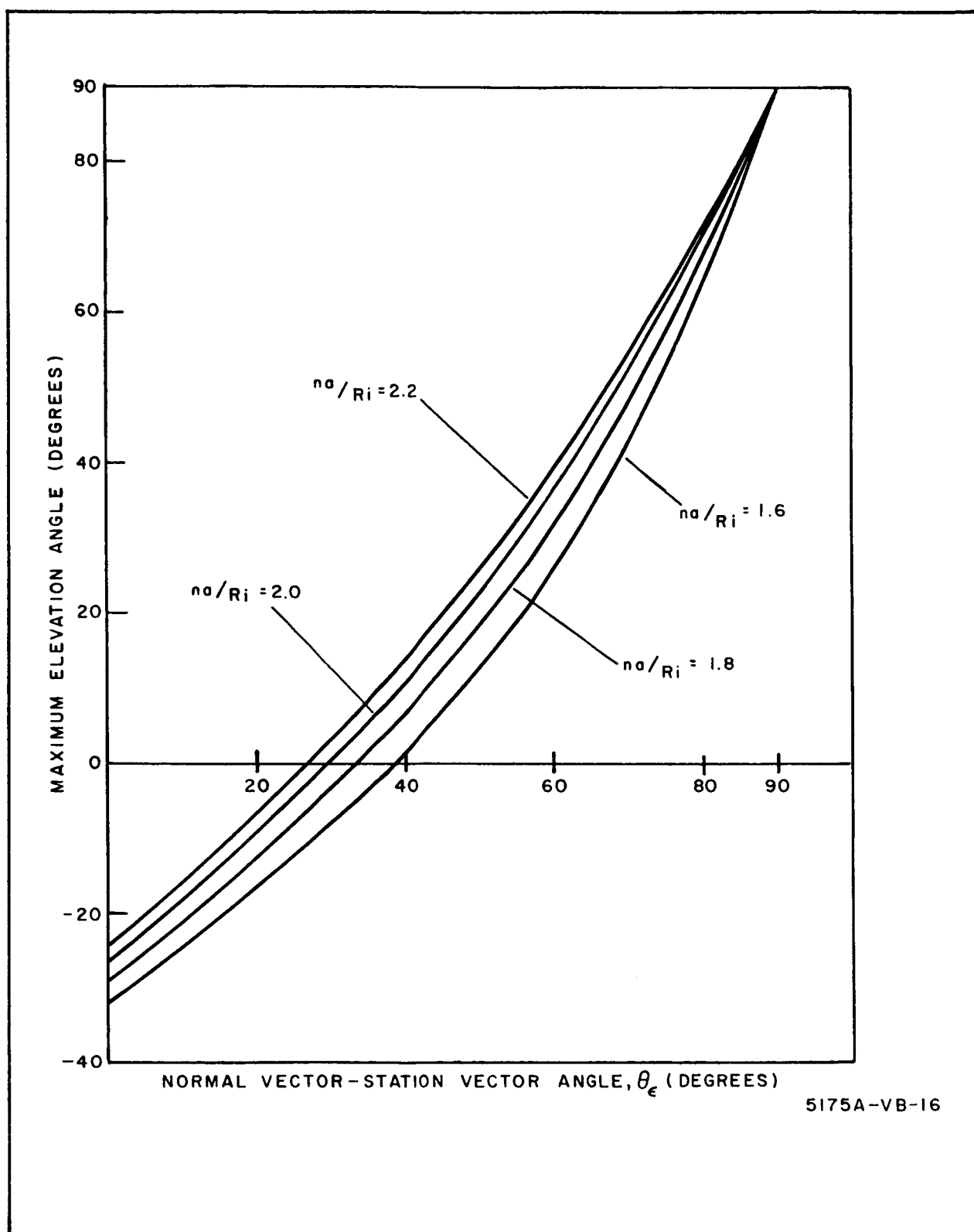
Figure 28.   Maximum Possible Elevation Angle

5175A-VB-16
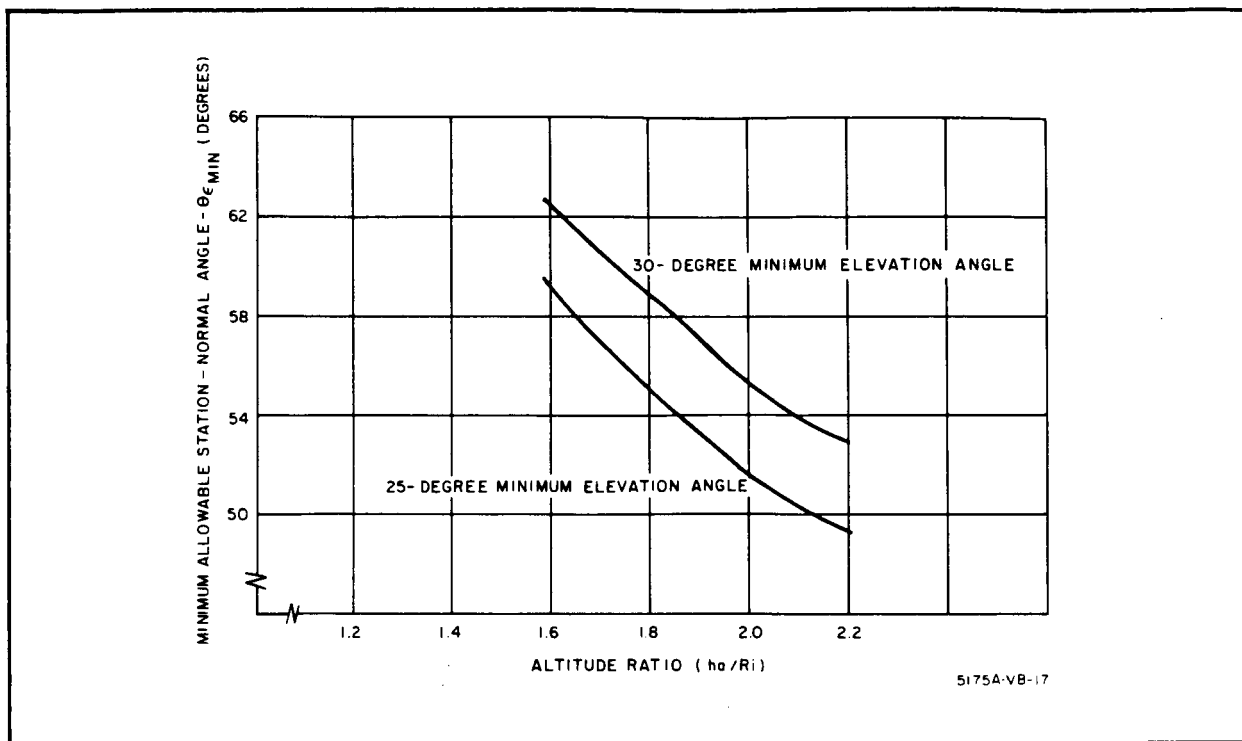
Figure 29. Minimum Allowable Station-Normal Angle

It can be seen from this expression that the maximum rate of change of the station-normal vector angle is approximately equal to the rotation rate of the earth. In addition, it has been found that the estimated maximum elevation times have always been accurate to at least ±1/8 of an orbit. At the altitudes of interest, this represents a time uncertainty of approximately 24 minutes. During this time the earth will rotate approximately 6 degrees. For this reason a safety factor of 6 degrees has been added to the minimum allowable station-normal vector angle. Thus, for the 25-degree minimum elevation angle requirement, the minimum allowable station-normal vector angle varies between 53 and 43 degrees as the altitude ratio varies between 1.6 and 2.2. A quadratic has been fit to this curve as a function of the altitude ratio which gives for the minimum allowable station-normal vector angle:

$$\theta_{\epsilon \ min} = 9 \ (h_a/R_i)^2 - 51.4 \ (h_a/R_i) + 112.4$$

The station-normal vector angle can be evaluated at the estimated maximum elevation time from the dot product of the station vector and the normal vector and is:

$$\cos \theta_\epsilon = \left| \cos \phi_i \sin i \sin (\theta_i - \Omega_T) - \sin \phi_i \cos i \right|$$

Then if:

$$\cos \theta_\epsilon \geq \cos \theta_{\epsilon_{min}}$$

the elevation angle cannot possibly be above 25 degrees and the more detailed computation need not be made for that station. It should be noted that this test is most effective for near equatorial stations and in fact will always fail for stations above the latitude:

$$\theta_{\epsilon_{min}} + (90 - i)$$

APPENDIX VI

ESTIMATION OF MAXIMUM ELEVATION TIME

A fairly good approximation to the maximum elevation time (figure 30) can be obtained by projecting the station vector into the plane of the orbit and calculating the time at which the satellite passes that point. For purposes of obtaining the first approximation for an orbit, the orbital elements are used as calculated at the half-orbit time in the manner described in Appendix V.

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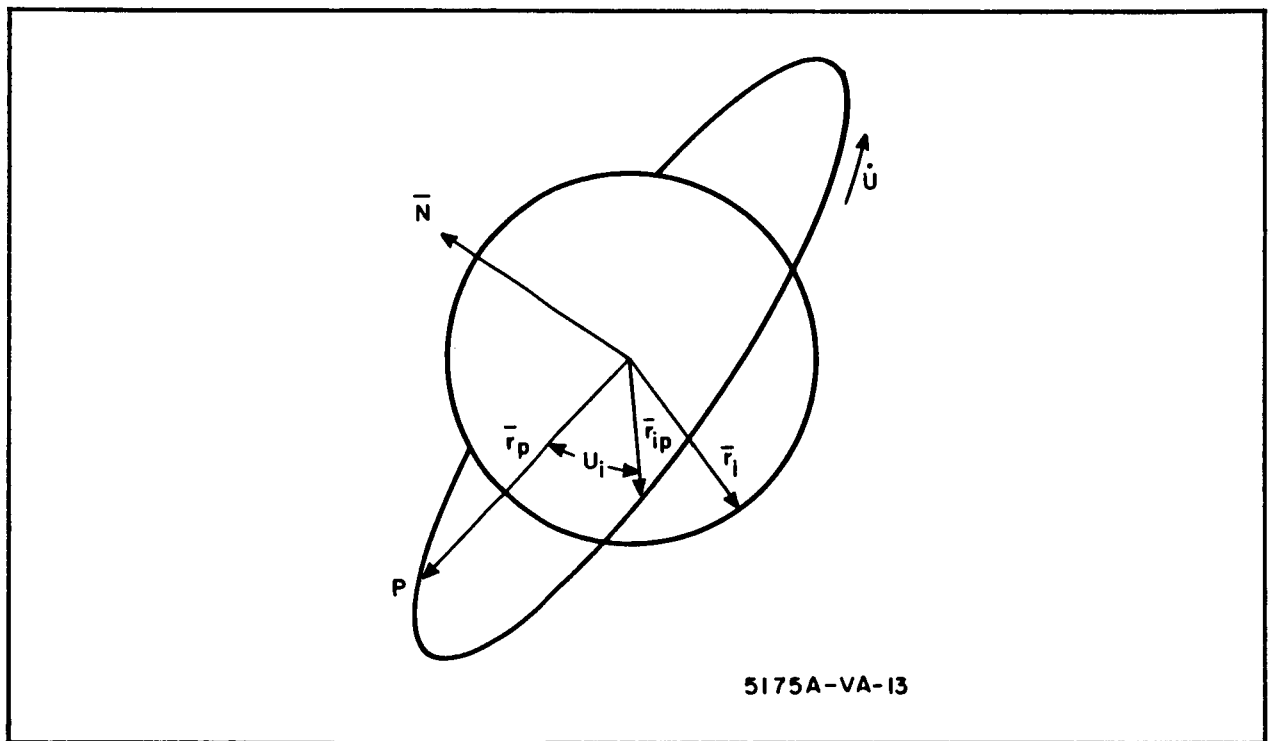
Figure 30. Geometry of Maximum Elevation Time Estimation

The station vector, $\overline{r}_i$, and a vector, $\overline{N}$, normal to the plane of the orbit can be expressed as:

$$\bar{r}_i = \begin{bmatrix} \cos\phi_i \cos\theta_i \\ \cos\phi_i \sin\theta_i \\ \sin\phi_i \end{bmatrix} \qquad \bar{N} = \begin{bmatrix} -\sin i \sin\Omega_T \\ \sin i \cos\Omega_T \\ -\cos i \end{bmatrix}$$

Then the projection of the station vector in the plane of the orbit is:

$$\bar{r}_{ip} = (\bar{N} \times \bar{r}_i) \times \bar{N}$$

A unit vector in the direction of the perigee point of the orbit can be expressed as:

$$\bar{r}_p = \begin{bmatrix} \cos\Omega_T \cos\omega - \sin\Omega_T \cos i \sin\omega \\ \sin\Omega_T \cos\omega + \cos\Omega_T \cos i \sin\omega \\ \sin i \sin\omega \end{bmatrix}$$

The true anomaly of the satellite, u_i, when it passes the projected station vector can be determined with four-quadrant resolution by the following manipulation of the vectors.

$$u_i = \tan^{-1}\left[\frac{-(\bar{r}_p \times \bar{r}_{ip}) \cdot \bar{N}}{\bar{r}_p \cdot \bar{r}_{ip}}\right]$$

The corresponding eccentric and mean anomalies are:

$$E_i = \tan^{-1}\left[\frac{\sqrt{1-e^2}\sin u_i}{e + \cos u_i}\right]$$

$$M_i = E_i - e \sin E_i$$

Then the estimated maximum elevation time is:

$$t_{est} = \frac{M_i}{n} + T_o$$

where T_o is the time of perigee passage. In practice, the estimated time is forced into the orbit under consideration by adjusting the mean anomaly, M_i, by an integral multiple of 2π. The estimated maximum elevation time is subsequently used as the starting value for the iteration procedure described in Appendix IV.

APPENDIX VII
VERTICAL PLANE ANGLE DETERMINATION

To obtain the desired accuracy in the determination of baseline distances, at least two of the observations for that baseline must be separated by a 60-degree angle with respect to a vertical plane through the baseline. The geometry of this angle is shown in figure 31. The angle, β, is defined as the angle between a vertical plane through the baseline and a plane containing the baseline and the satellite. This angle can be determined by calculating two vectors, $\overline{V}_1$ and $\overline{V}_2$, perpendicular to the two planes. If the radius vector to the satellite, $\overline{r}$, and vectors to the two stations defining the baseline, $\overline{r}_i$ and $\overline{r}_j$, are known, the angle β can be determined as follows:

$$\overline{r}_{ij} = \overline{r}_j - \overline{r}_i \quad \text{(vector between two stations)}$$

$$\overline{r}_{is} = \overline{r} - \overline{r}_i \quad \text{(vector from station i to satellite)}$$

$$\overline{V}_1 = \overline{r}_{ij} \times \overline{r}_{is} \quad \text{(vector perpendicular to baseline, satellite plane)}$$

$$\overline{V}_2 = \overline{r}_{ij} \times \overline{r}_i \quad \text{(vector perpendicular to vertical plane through baseline)}$$

$$\beta = \cos^{-1} \left[\frac{\overline{V}_1 \cdot \overline{V}_2}{V_1 \, V_2} \right]$$

To determine which side of the vertical plane the satellite lies on, a third vector, $\overline{V}_3$, in the plane of $\overline{V}_1$ and $\overline{V}_2$ and perpendicular to $\overline{V}_2$ can be determined by:

$$\overline{V}_3 = \overline{V}_2 \times \overline{r}_{ij}$$

Then if:

$$\overline{V}_1 \cdot \overline{V}_3 < 0$$
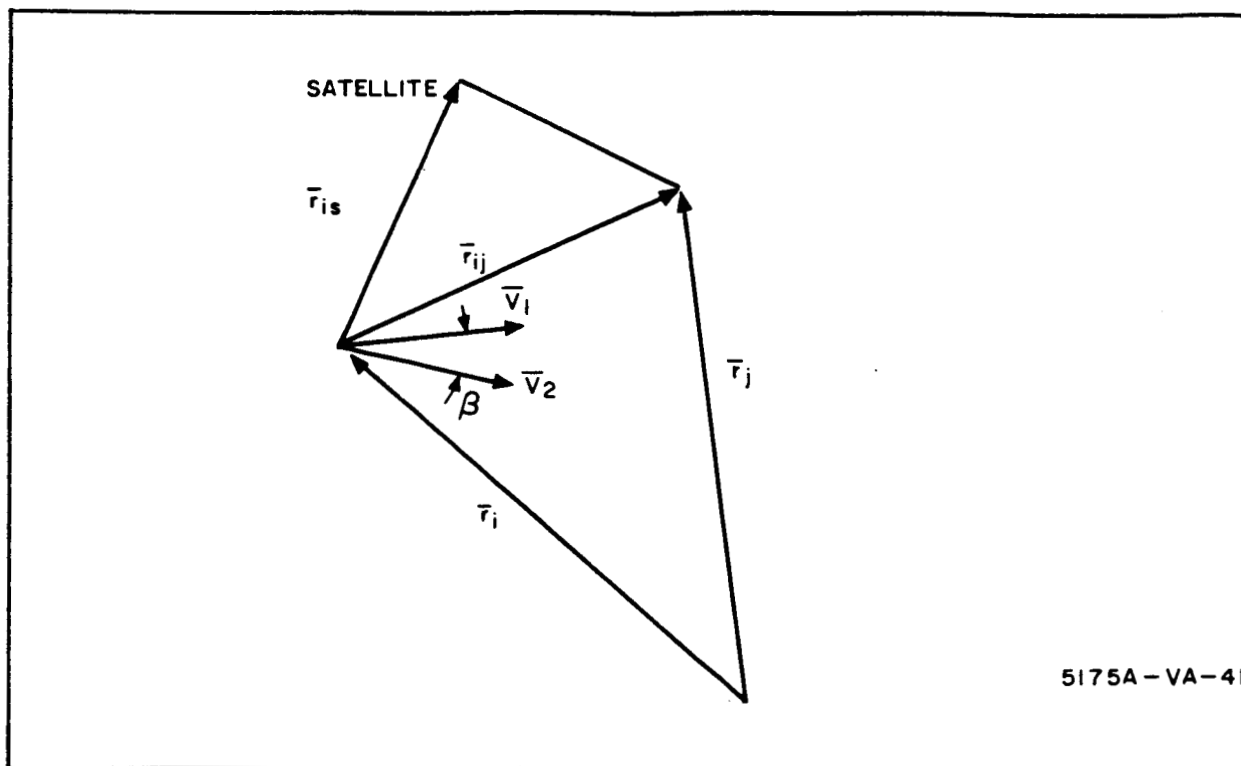
Figure 31. Vertical Plane Angle Geometry

The satellite lies to the right of the vertical plane when looking from station i to station j and the angle β is defined as negative. Otherwise, the satellite lies to the left of the vertical plane when looking from station i to station j and the angle β is defined as positive.

APPENDIX VIII
RESULTS FROM ABBREVIATED RUNS, ORBITS 2 THROUGH 6

The following tables show the results obtained from the abbreviated runs (seven stations, 1 month in 3 time sample) for orbits numbered 2 through 6. The meanings of the various entries are explained in the section entitled "Discussion and Results" (Final Orbit Selection).

BASELINES		607	698	789	880	970	1062	1153	1244	1335	1426	1517	END	TOTAL					
		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
1-2	G		1											1					
	M		3				1							4					
	+		1											1					
	−																		
1-3	G	3		3								7		6					
	M			1				2						10					
	+																		
	−	5		4										9					
2-3	G	9		2		1							17	29					
	M	2									1		3	6					
	+																		
	−	6		2		2							13	23					
2-4	G	1		3										4					
	M	1		1				1			1			4					
	+																		
	−			3										3					
3-4	G						1							1					
	M																		
	+																		
	−																		
3-5	G	22	16		25	5	1		23	13			21	126					
	M	8	1		2	2	1		5	7	8		1	35					
	+	3				3				8				14					
	−	14	3		10				8					35					
4-5	G	4	8		15	4	5		6					42					
	M	2			1	5	4		1					13					
	+		1		7	1								9					
	−	3				2								5					
4-7	G	18	9		8	17	10		17	1			15	83					
	M	4	3			6				3				26					
	+	2	3		6					1				12					
	−	4			1	14							4	23					
5-6	G	38	14		4	47	22			46	3			174					
	M																		
	+	68	14		4	75	12			46				219					
	−		7		4		12			18	3			44					
5-7	G	32	3			14	7			23				79					
	M	1									2			3					
	+						3							3					
	−	43	3			26	7			38				117					
6-7	G	18				21				7				46					
	M																		
	+																		
	−																		
TOTAL	G													590					
	M													102					
	+																		
	−																		

5175A-VC-1

ORBIT 2: 4250 km, 89° INCLINATION
Ω = 10°, JUNE 1 1966 LAUNCH

TRIANGLES AND BASELINES		607	698	789	088	970	1062	1153	1244	1335	1426	1517	END	TOTAL
END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965														
1 2 3	G													
	M													
	+													
	−													
	+													
	−													
	+													
	−													
2 3 4	G	1												1
	M													
	+	1												1
	−													
	+													
	−	2												2
	+													
	−	1												1
3 4 5	G	1	9		8				7					25
	M		3		2				2					7
	+													
	−	1	9		8				7					25
	+		7		7				8					22
	−													
	+													
	−	1	1		2				3					7
4 5 7	G	1	7		6	12				5				14
	M	3	8		6									26
	+													14
	−													
	+													
	−	2												2
	+	1	7		6									14
	−	1	6		6									13
5 6 7	G	20				18			10					48
	M	4												4
	+	21				28			20					69
	−	14												14
	+									3				3
	−	39				29				10				78
	+													
	−	3												3
TOTAL	G													80
	M													38
	+													
	−													
	+													
	−													
	+													
	−													

5175A-VC-2

ORBIT 2 : 4250 km , 89° INCLINATION
Ω = 10° , JUNE 1 1966 LAUNCH

BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1,1965												
		607	698	789	880	970	1062	1153	1244	1335	1426	1517	END	TOTAL
1-2	G		3											3
	M		3											4
	+		1	1										1
	−													
1-3	G	2		4	2			4						10
	M			2										4
	+													
	−	2		5				4						11
2-3	G	11						2		4	1			18
	M									1				1
	+													
	−	11						3		4			2	20
2-4	G	1		4		7		3						15
	M	1		2		1		2		1				7
	+													
	−	1		2		12		2						17
3-4	G													
	M													
	+													
	−													
3-5	G	21	14		21	2	3		17	7	1		11	96
	M	6			2	3	16		7	1			12	48
	+	5				4			1	6				16
	−	11	1		8				2					22
4-5	G	3	9		19	2	13		10	5	8		8	67
	M	3	3		5		4		5	3			3	33
	+				12				1	7				23
	−	3												3
4-7	G	18	9			9	5	8	4	7	11		7	67
	M	3	2					6	1	2			2	27
	+	1	7			5				3				16
	−	6												6
5-6	G	36	13				25		6		20		5	105
	M		1				1						1	3
	+	65	13				23		5		15		5	126
	−		7				17		6		11		1	42
5-7	G	31	3		5	7	3		1		8		3	61
	M						1				4			5
	+													
	−	41	3			3	3		1		8		1	60
6-7	G	16				2								18
	M			.										
	+													
	−	1				2								3
TOTAL	G													460
	M													127
	+													
	−													

ORBIT 3: 4500 km, 90° INCLINATION
Ω = 10°, JUNE 1 1966 LAUNCH

5175A-VC-1

TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1,1965																	
		607	698	789	880	970	1062	1153	1244	1335	1426	1517	END	TOTAL					
1 2 3	G M + -																		
	+ - + -																		
2 3 4	G M + -	1		1										1 1					
	+ - + -	2 1												2 1					
3 4 5	G M + -	2 2	10 1 8		10 2 10				5					22 8 20					
	+ - + -	1 2	12 3		9 3									22 8					
4 5 7	G M + -	4	6 6 9		1 1	1 4			3 1 2	8	2			11 25 12					
	+ - + -		6 6		1	1 1			3 3					11 10					
5 6 7	G M + -	19 6 19 12												19 6 19 12					
	+ - + -	38 7												38 7					
TOTAL	G M + -													55 39					
	+ - + -																		

5175A-VC-2

ORBIT 3 : 4500 km , 90° INCLINATION
Ω = 10° , JUNE 1 1966 LAUNCH

BASELINES		607	698	789	880	970	1062	1153	1244	1335	1426	1517	END	TOTAL						
1-2	G				1									1						
	M		2				2							4						
	+				1									1						
	−																			
1-3	G	1			2	9	8	2		3	3	6		31						
	M				2			6		2				13						
	+					1								1						
	−	1			2	2	5	1		1		4		16						
2-3	G	9		5	2	9	13	1	1	4	3	5		52						
	M					1		2	2		1	1		7						
	+			5		2			1					8						
	−	9				3	11			2		6		31						
2-4	G				3	6	5			2		5		21						
	M					3						3		6						
	+				2									2						
	−						1					3		4						
3-4	G				4			5						9						
	M		1											1						
	+																			
	−							1						1						
3-5	G	24	20		23	8	13		2	1		2	1	93						
	M	5	6		3	1	5		3		13		3	39						
	+	5				2	1							8						
	−	9	3		11	2	6							31						
4-5	G	3			3	3			4					13						
	M	3	1		1				4		6		5	20						
	+								3					3						
	−	1			2									3						
4-7	G	15	14		18	4						2		53						
	M	5	4			8								13						
	+				4									8						
	−				2									2						
5-6	G	46	21		9	39			7	25	7	4		158						
	M		7		1					1	1			3						
	+					23			8	4	8	7		141						
	−	75	15		9	9				5				29						
5-7	G	34	3		3	36			5	19		3		103						
	M	1	1											2						
	+									4				4						
	−	37	3		2	68			6	18		2		136						
6-7	G	1												1						
	M																			
	+																			
	−	1												1						
TOTAL	G													535						
	M													101						
	+																			
	−																			

5175A-VC-1

ORBIT 4.   4000 km , 85° INCLINATION

Ω = 10° , JUNE 1 1966 LAUNCH

TRIANGLES AND BASELINES		607	698	789	880	970	1062	1153	1244	1335	1426	1517	END	TOTAL				
1 2 3	G				3		1							4				
	M						1							1				
	+													2				
	−				2									4				
	+				3		1							4				
	−																	
	+				4		2							6				
	−																	
2 3 4	G	1			4		2							6				
	M					1	2	4		1				9				
	+				4		3							7				
	−																	
	+													3				
	−				3													
	+				2	2								4				
	−				4									4				
3 4 5	G	1	4		7		1	1						12				
	M		3		1	1								5				
	+				7									1				
	−	1	5		7									13				
	+	1	4		5									10				
	−																	
	+																	
	−	1	1		5									7				
4 5 7	G	2	12		1	1						1		17				
	M	4	7		3	5								19				
	+		5											5				
	−																	
	+													7				
	−	2	3		2	1								17				
	+	2	12		1	1						1		13				
	−		11		1	1												
5 6 7	G	43				43					3			87				
	M	1												1				
	+	48				77					3			128				
	−	15									2			17				
	+					9								9				
	−	73				44					6			123				
	+					5								5				
	−																	
	G													129				
	M													38				
	+																	
	−																	
	+																	
	−																	
	+																	
	−																	

5175A-VC-2

ORBIT 4: 4000 km, 85° INCLINATION
Ω = 10°, JUNE 1 1966 LAUNCH

Baselines		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965												
		607	698	789	880	970	1062	1153	1244	1335	1426	1517	END	TOTAL
1-2	G		3			2					1			3
	M					1					2			6
	+					1								1
	−													
1-3	G		1	9	2		6	14		10	8	19		64
	M							1			1	1		3
	+		2				2			5				9
	−		1		1			12			7	11		32
2-3	G	5		17	3		13	1		5	6	9	1	60
	M							1		1		2		4
	+	2		10			7							21
	−	3		3	3		3				5	1	2	19
2-4	G	1	1	3	7		4	13		7	6	4		44
	M													7
	+				2		1	2		3				4
	−				6		1	4			1	1		11
3-4	G		8		2				1					10
	M													1
	+		1											1
	−		6		2									8
3-5	G	27	25				10		24	12	24		2	129
	M	4	3				5		3	6	1		2	24
	+	10					8			6			4	28
	−	10	10						12		11			43
4-5	G	4	3		1		4	7		1				20
	M	3			1		2			1			1	8
	+	2			2		2							6
	−		1											1
4-7	G	20	22		10	1			11	9	25			98
	M	2	3		2					4	1			12
	+	4	5								2			12
	−	4	5		9				1		5			23
5-6	G	40	7			37				9	19			113
	M													
	+	12								9	20			41
	−	2	7			8					9			26
5-7	G	30				26					9			65
	M													
	+					2								2
	−	3				26								29
6-7	G									29				29
	M													
	+													
	−													
TOTAL	G													601
	M													84
	+													
	−													

5175A-VC-1

ORBIT 5: 4500 km 80° INCLINATION
$\Omega = 0°$, JUNE 1 1966 LAUNCH

TRIANGLES AND BASELINES		607	698	789	880	970	1062	1153	1244	1335	1426	1517	END	TOTAL				
1 2 3	G			1			1	4				1		7				
	M																	
	+																	
	−							1						1				
	+						1	3				1		5				
	−																	
	+																	
	−			2			2	6				2		12				
2 3 4	G		1			1		2		1	1	7		13				
	M					2		2		1	3	3		9				
	+		2					3		1	1	9		18				
	−																	
	+																	
	−							1			1	5		7				
	+		1							1		4		6				
	−							2				7		10				
3 4 5	G		6						6		5			17				
	M					1			4	2	4			11				
	+																	
	−		5						5		6			16				
	+		4						9		4			17				
	−																	
	+																	
	−		4						2		3			9				
4 5 7	G	4	5			2			1		6			19				
	M	9	5			4			3	1	6			28				
	+	3				2				1	4			10				
	−																	
	+													4				
	−													21				
	+	4	7			2			1	1	3			8				
	−	3	3			2					6							
5 6 7	G	40				9								49				
	M																	
	+	40												40				
	−	14												15				
	+	5												5				
	−	40				9								49				
	+					9								9				
	−																	
TOTAL	G													107				
	M													48				
	+																	
	−																	
	+																	
	−																	
	+																	
	−																	

5175A-VC-2

ORBIT 5 : 4500 km   80° INCLINATION
Ω = 0°   , JUNE 1 1966 LAUNCH

BASELINES		729	820	910	1002	1092	1184	1275	1366	1461	1548	1639	1668	TOTAL
1-2	G			1								1	3	3
	M													2
	+													
	-												1	1
1-3	G	8	7	7	9			10		3	5		4	53
	M									1	1			2
	+	5	2	2				6			1			16
	-		4	.3	11						1			19
2-3	G	12	10	5	11				1		12	10	6	66
	M										2			3
	+	9									6	1		16
	-	1	6	1	7						1	3		19
2-4	G	6	11	3	2			8			6	3	11	50
	M		1	1								3		10
	+	1	1		1			2			2		3	11
	-	3	3		3			2					1	7
3-4	G			1	1	1				1	1	9		13
	M					1				1		1		3
	+											4		4
	-		1		1							9		12
3-5	G	2		19	13	12		2	11		22		2	79
	M			6	3	4			3		6			26
	+			1	10						2			13
	-			2	8						6			16
4-5	G	9		10	20	10			8		9		44	105
	M	5		7	3	6			2				9	30
	+	5		1									20	28
	-				10								7	17
4-7	G			3	32			1	20	6	13		38	112
	M			3	7				1		14		4	26
	+				1				2				4	7
	-				16								16	32
5-6	G			41				30			28			99
	M			4							1			5
	+			52				31			41			124
	-			7				17			5			29
5-7	G					13		32			36			81
	M										1			1
	+							15			1			16
	-							14			42			56
6-7	G							1						1
	M													
	+													
	-							1						1
TOTAL	G													662
	M													108
	+													
	-													

5175A-VC-1

ORBIT 6: 4250 km, 87° INCLINATION
Ω = 80°, OCT 1 1966 LAUNCH

TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		729	820	910	1002	1092	1184	1275	1366	1461	1548	1639	1668	TOTAL					
1 2 3	G	1									5	4		11					
	M +	1									2	1		4					
	−	1									2	2		5					
	+	1									5	5		11					
	− +																		
	−	1	2								7	4		14					
2 3 4	G		3	1	5								1	10					
	M		2	2	2								1	7					
	+		3		6									11					
	−		3											3					
	+		1		4									7					
	−		2	1	3							2		6					
	+		3		5							1		9					
3 4 5	G			1	6	4								7					
	M			4	3							4		12					
	+			1	6									3					
	−													7					
	+			2	6									8					
	− +																		
	−			1	4									5					
4 5 6	G			4										4					
	M			4										5					
	+			2				1						2					
	−																		
	+			4										4					
	− +																		
	−			4										4					
5 6 7	G			4				5				3		12					
	M			8				6				4		18					
	+											1		1					
	−																		
	+			4										4					
	−			4				5				6		15					
	+							4				1		5					
	−			4				5						9					
TOTAL	G													44					
	M +													24					
	−																		
	+																		
	− +																		
	−																		

5175A-VC-2

ORBIT 6: 4250 km , 87° INCLINATION
Ω = 80° , OCT 1 1966 LAUNCH

APPENDIX IX

GRAPHICAL ANALYSIS

Introduction

To obtain preliminary information concerning the viewing conditions to be expected at polar stations, a graphical analysis was made. As it appeared that conditions were more critical at the south pole, effort was restricted to the three stations nearest that pole. Circular orbits were assumed and the only perturbation considered was the regression of the orbit ascending node.

Results and conclusions of the analyses are presented in the next section, while later sections contain descriptions of the graphical methods used in obtaining these results.

Results and Conclusions

Darkness. - Figure 32 illustrates the periods of darkness for the various south pole station pairs, as a function of season of the year. (A condition of darkness is taken to exist at a station whenever the sun is more than 18 degrees below the horizon.) S_o is the angle from the Greenwich meridian to the vernal equinox, positive eastward. Note that at no time are three stations simultaneously in darkness, thus prohibiting a simultaneous three-station observation from stations of the south pole triangle.

Viewing Opportunities Versus Orbit Plane Orientation. - Figures 33 and 34 illustrate the average number of viewing opportunities per 30-day period for a specific date (June 21), as a function of the longitude of the ascending node of the orbit. The stations considered were the pair 29 and 35, with figure 33 plotted for an orbit inclination of 80 degrees and figure 34 plotted for an 83-degree inclination. Orbit altitudes of 4000 and 5000 km were considered, as indicated on the graphs. Note that the 4000-km, 80-degree orbit is unsatisfactory since observations are not possible for $\lambda > 30$ degrees, precluding the
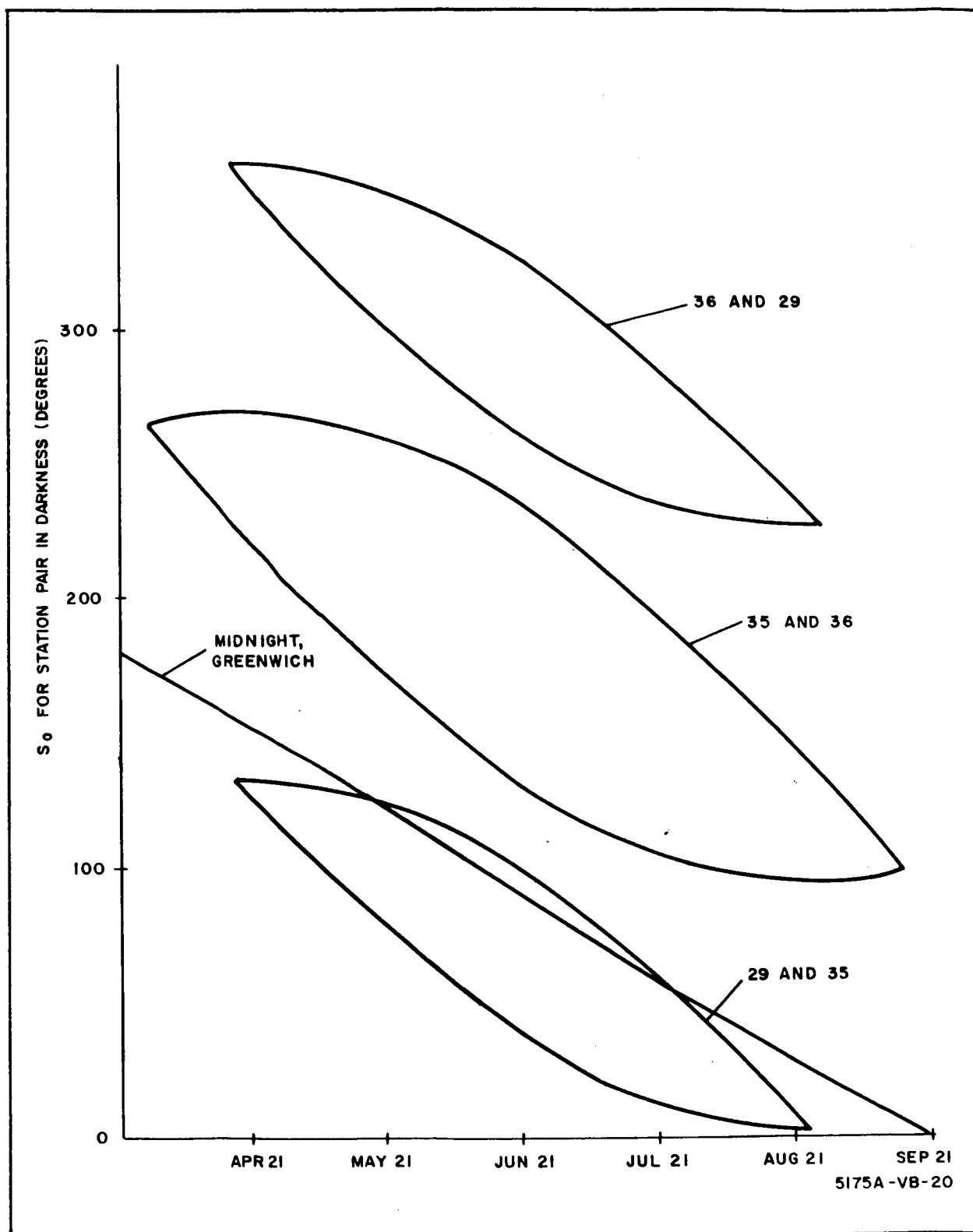
Figure 32. Darkness Periods, South Pole Stations
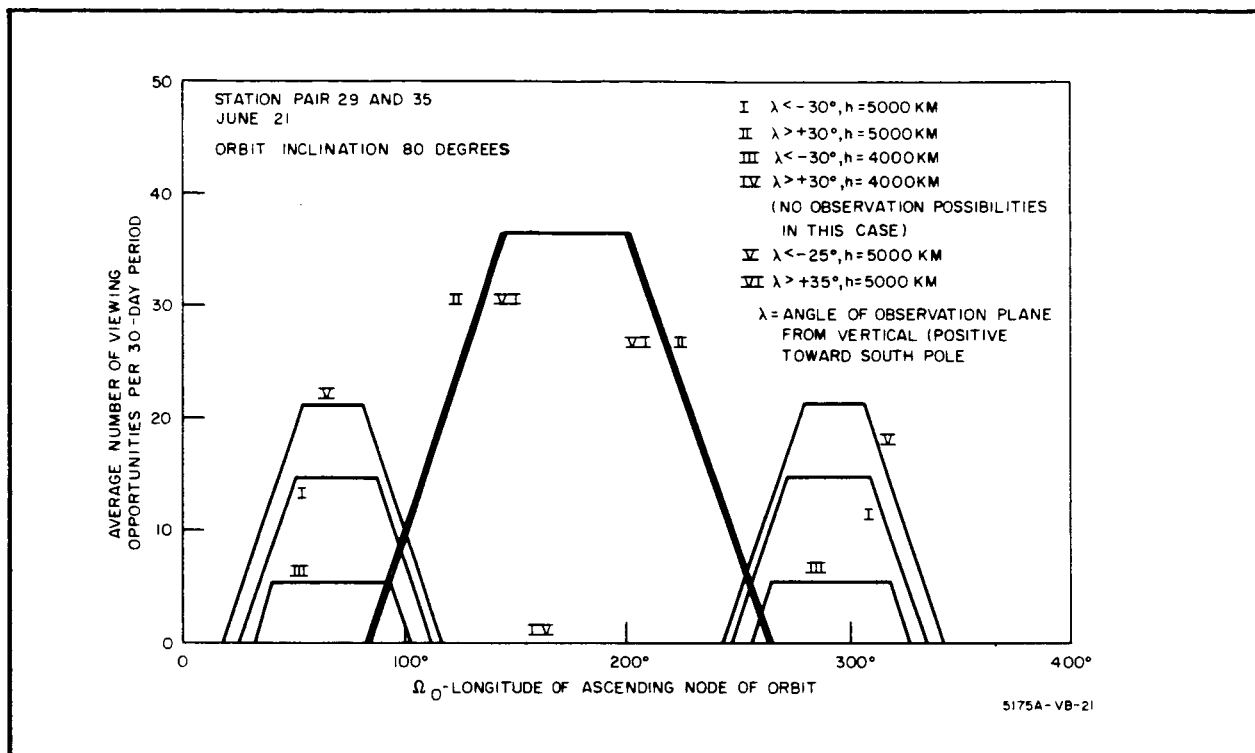
Figure 33. Fixed Date Observation Opportunities
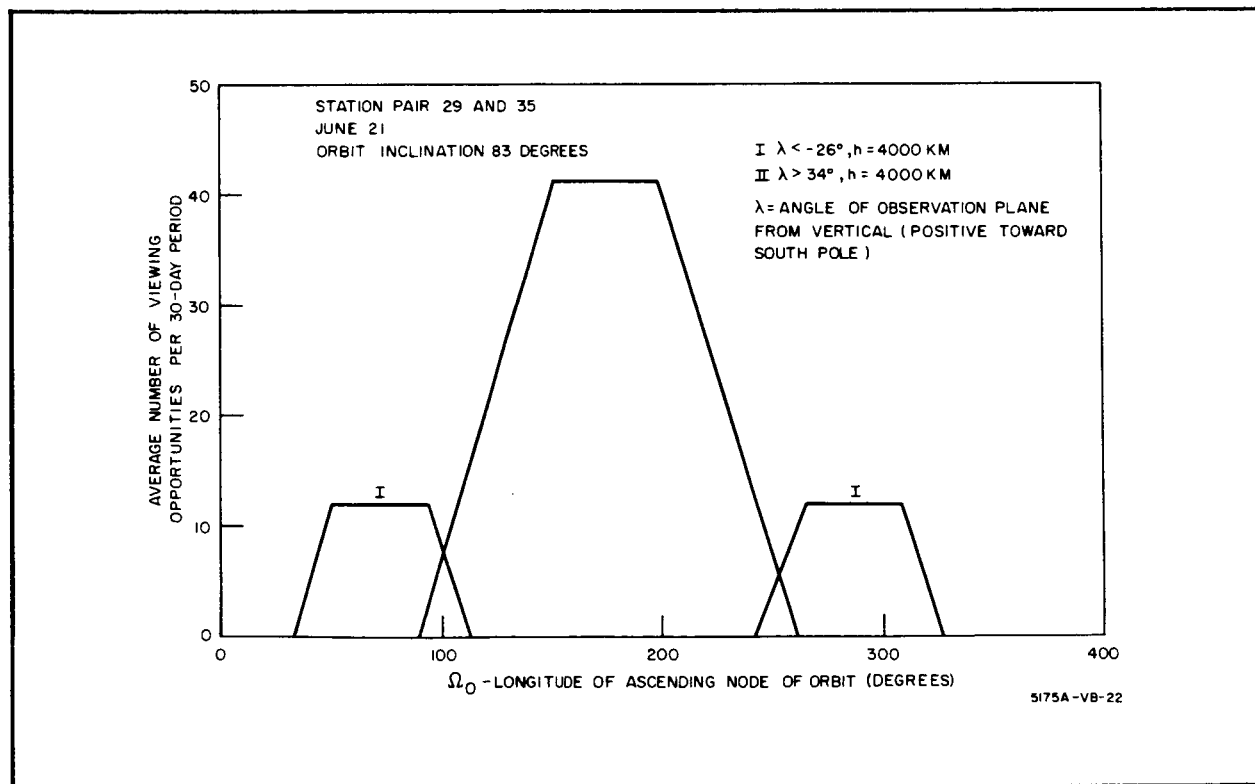
Figure 34. Fixed Date Observation Opportunities

possibility of obtaining observation planes intersecting at 60-degree or greater angles. Also note that various positions of the orbit plane are required to obtain observations at various observation plane angles.

Viewing Opportunities Versus Date. - The average number of viewing opportunities per 30-day period was determined as a function of date for various station pairs, as illustrated in figures 35 through 43. Various initial orbit plane orientations were chosen, circular orbits were assumed, and orbit perturbations other than the regression of nodes were ignored. Orbit inclinations of 83 and 90 degrees were chosen for study, since the 80-degree orbits appear of marginal value, and because orbits in the range of about 84 to 86 degrees inclination appear from previous studies to have poor lifetimes. Significant items of note are:

a. For the 83-degree orbit, opportunities for high and low latitude observations occur at different times, extending the required station occupation times over what they otherwise might be.

b. With an initial longitude of the ascending node of 108 degrees west, observations at the station pair (29, 35) are satisfactory but observations at the station pair (35, 36) are not. (Compare figures 36 and 37.) For an initial longitude of 90 degrees east, observations at all station pairs are possible. (Figures 38, 39, and 40.) This illustrates the importance of the initial orbit plane orientation in obtaining satisfactory observations from the near-polar station pairs.

c. At a 90-degree orbit inclination, 4500-km altitude, viewing opportunities appear better for the station pairs near the south pole than for the 83-degree, 4000-km altitude. It should be pointed out, however, that although attempts were made to choose relatively favorable initial orbit plane orientations, those chosen are not necessarily optimum, and thus the above statement might be altered as a result of further study.

d. Although consideration of inflation requirements at launch was not included in this preliminary study, it may be noted that initial longitudes of the ascending node of the order of 90 degrees east or west in combination with

STATIONS 29 AND 35
83-DEGREE ORBIT INCLINATION, 4000-KM ALTITUDE
INITIAL CONDITIONS: LONGITUDE OF ASCENDING NODE OF ORBIT
118° WEST OF VERNAL EQUINOX ON APRIL 21
λ = ANGLE OF OBSERVATION PLANE FROM VERTICAL
(POSITIVE TOWARD SOUTH POLE)

5175A-VB-23
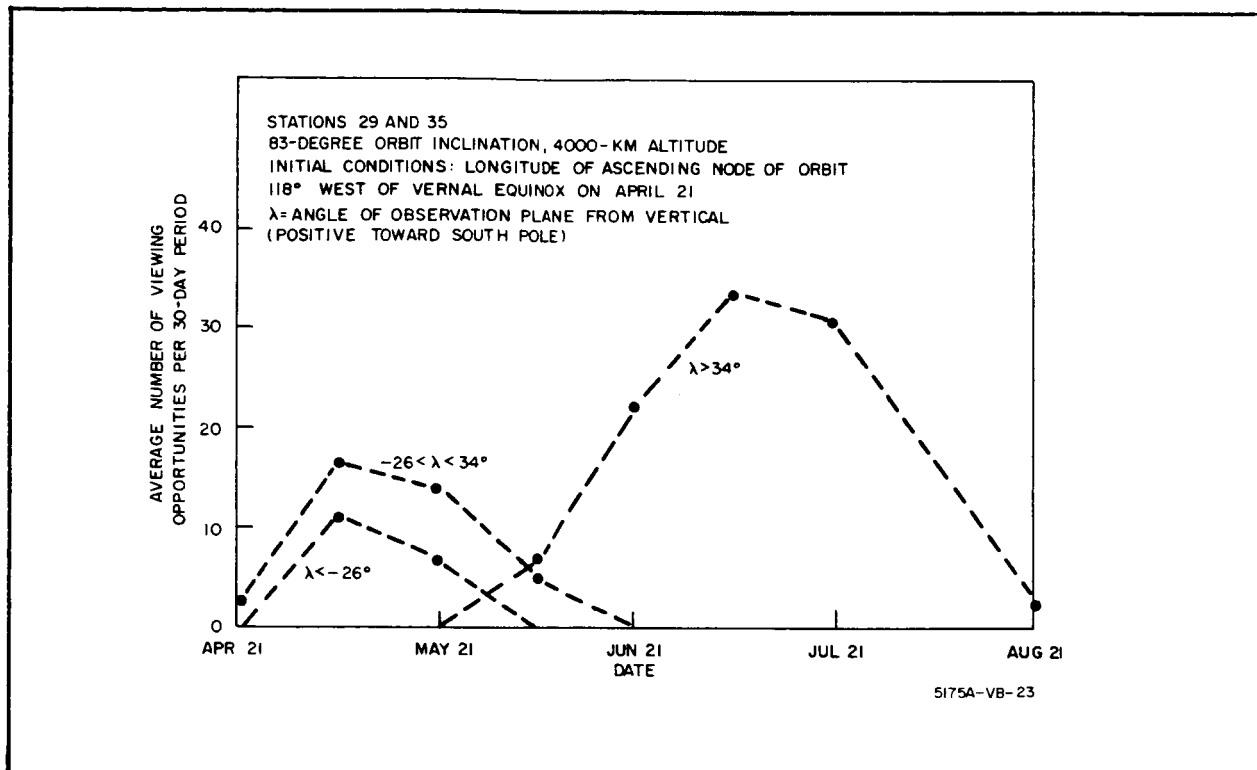
Figure 35. Observation Opportunities for Stations 29 and 35

STATIONS 29 AND 35
83-DEGREE ORBIT INCLINATION, 4000-KM ALTITUDE
INITIAL CONDITIONS: LONGITUDE OF ASCENDING NODE
OF ORBIT 108° WEST OF VERNAL EQUINOX ON APRIL 21
λ = ANGLE OF OBSERVATION PLANE FROM VERTICAL
(POSITIVE TOWARD SOUTH POLE)

5175A-VB-24
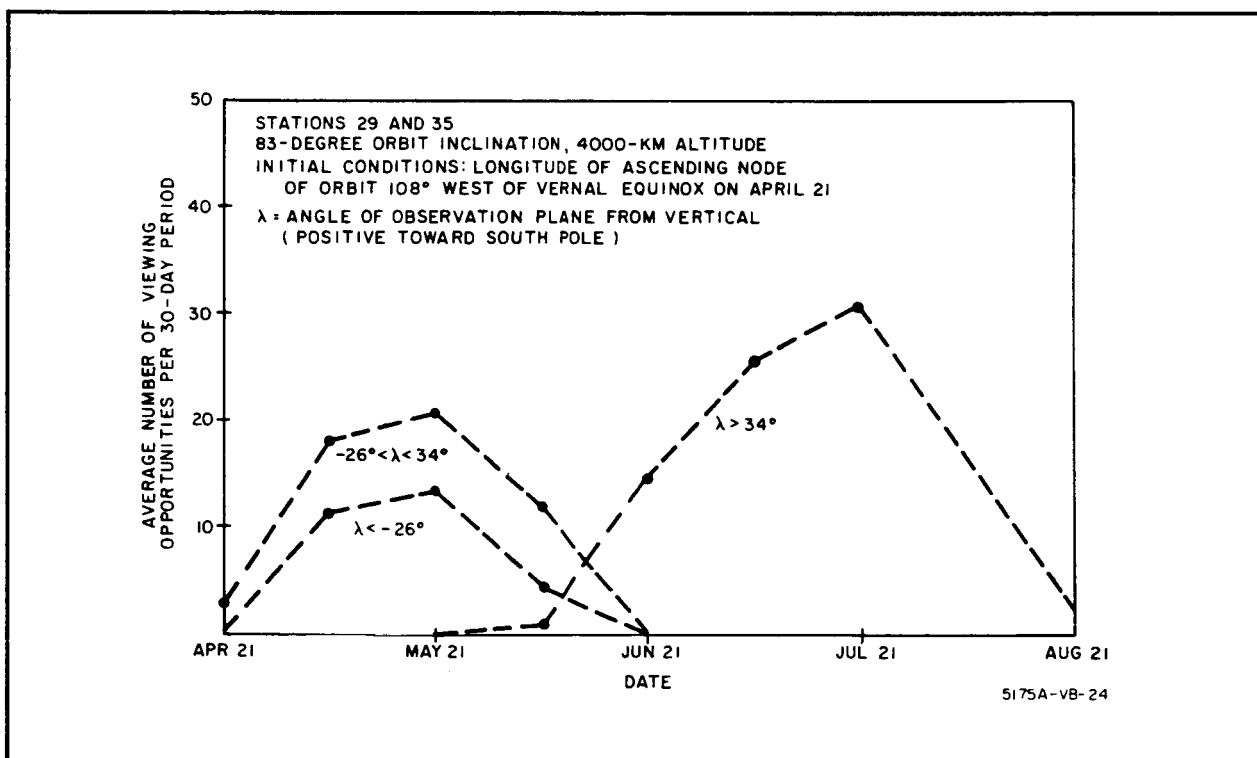
Figure 36. Observation Opportunities for Stations 29 and 35

IX-5
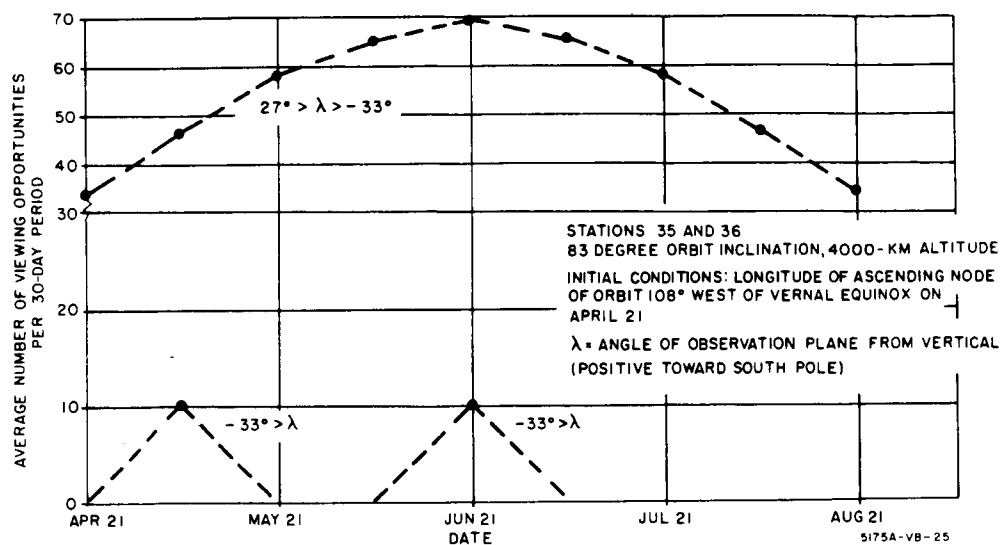
Figure 37.   Observation Opportunities for Stations 35 and 36
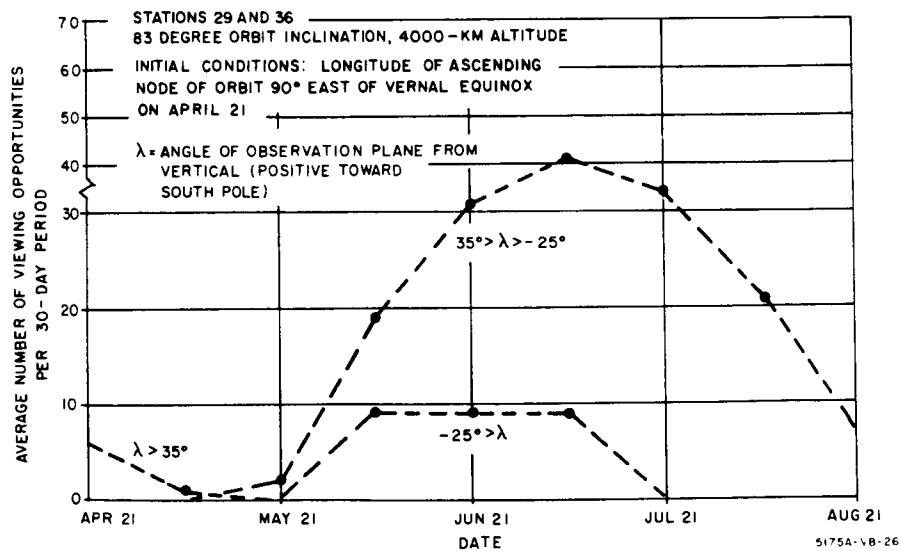
Figure 38.   Observation Opportunities for Stations 29 and 36
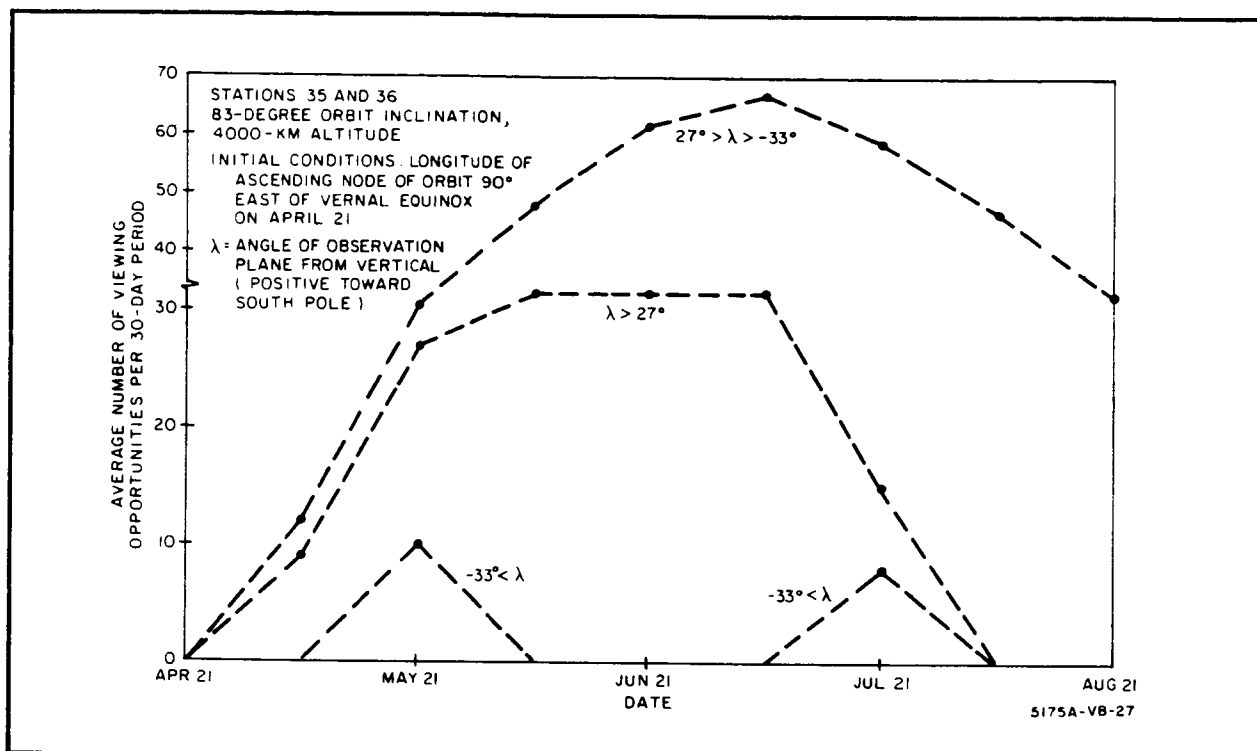
Figure 39. Observation Opportunities for Stations 35 and 36
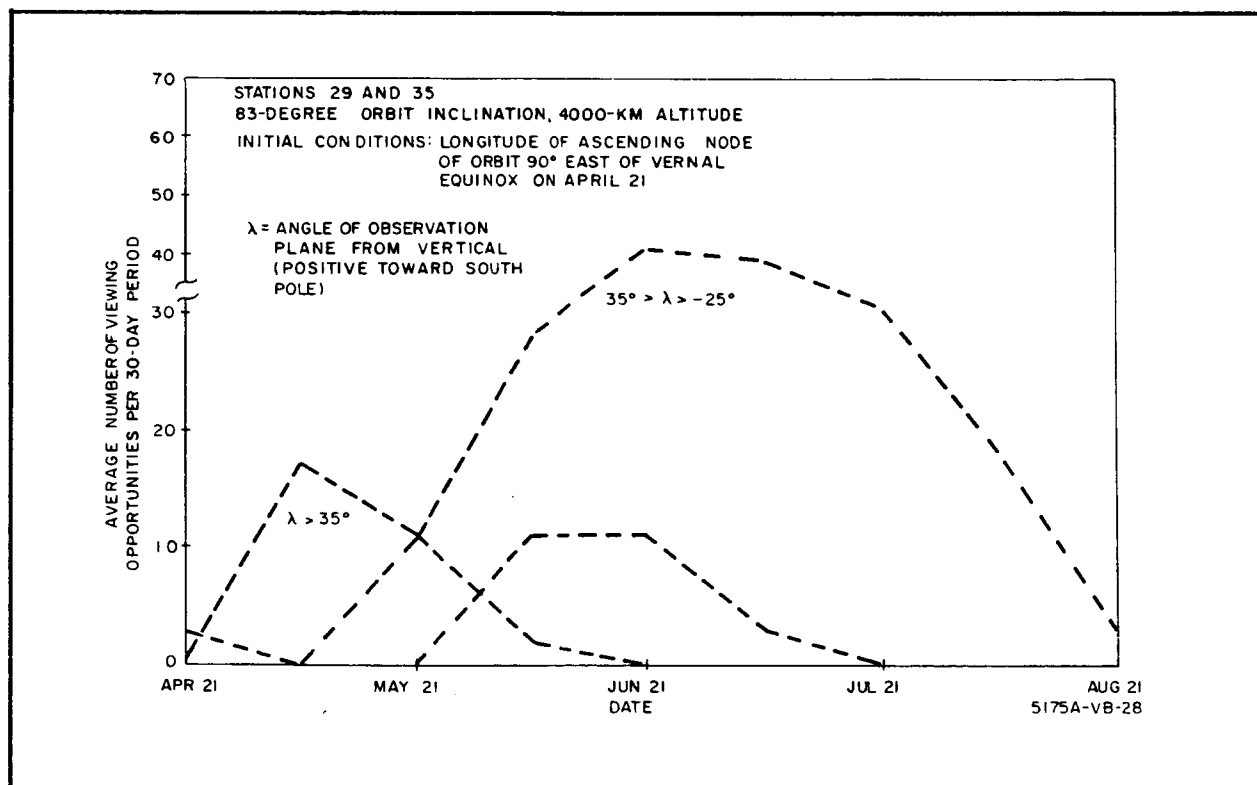
Figure 40. Observation Opportunities for Stations 29 and 35
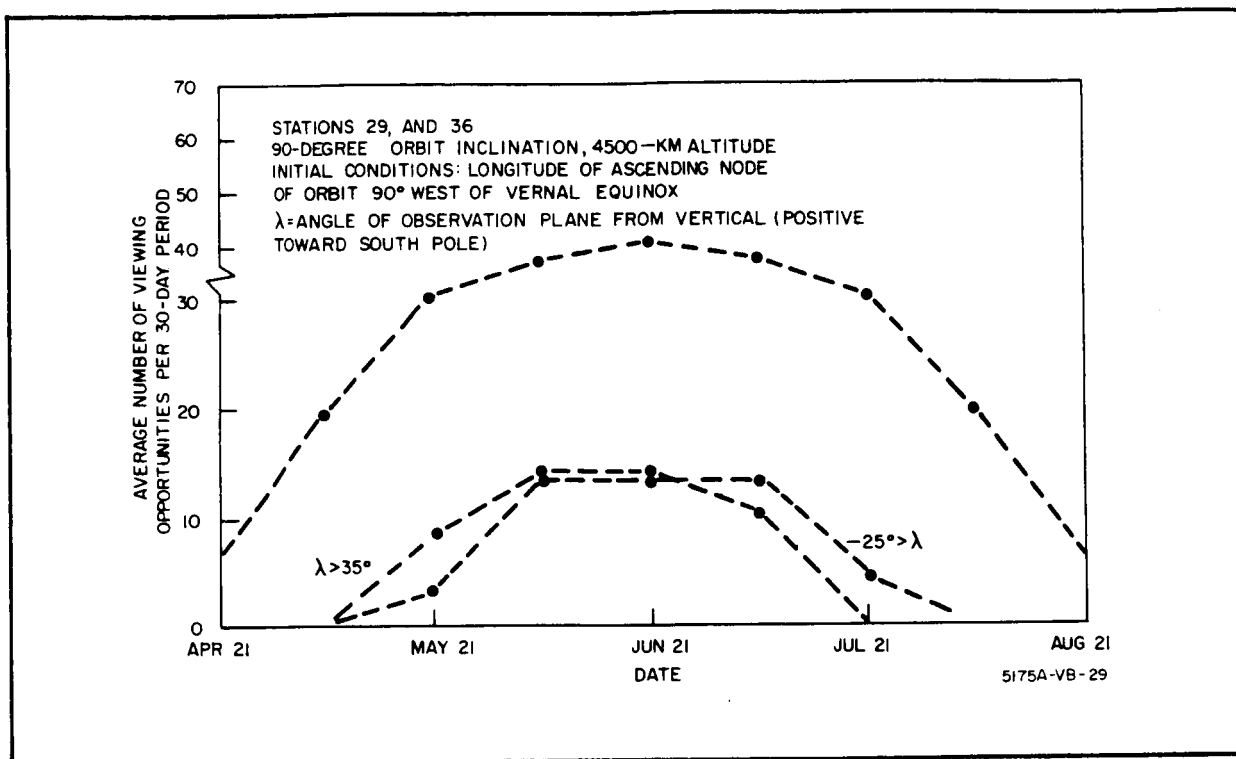
Figure 41. Observation Opportunities for Stations 29 and 36
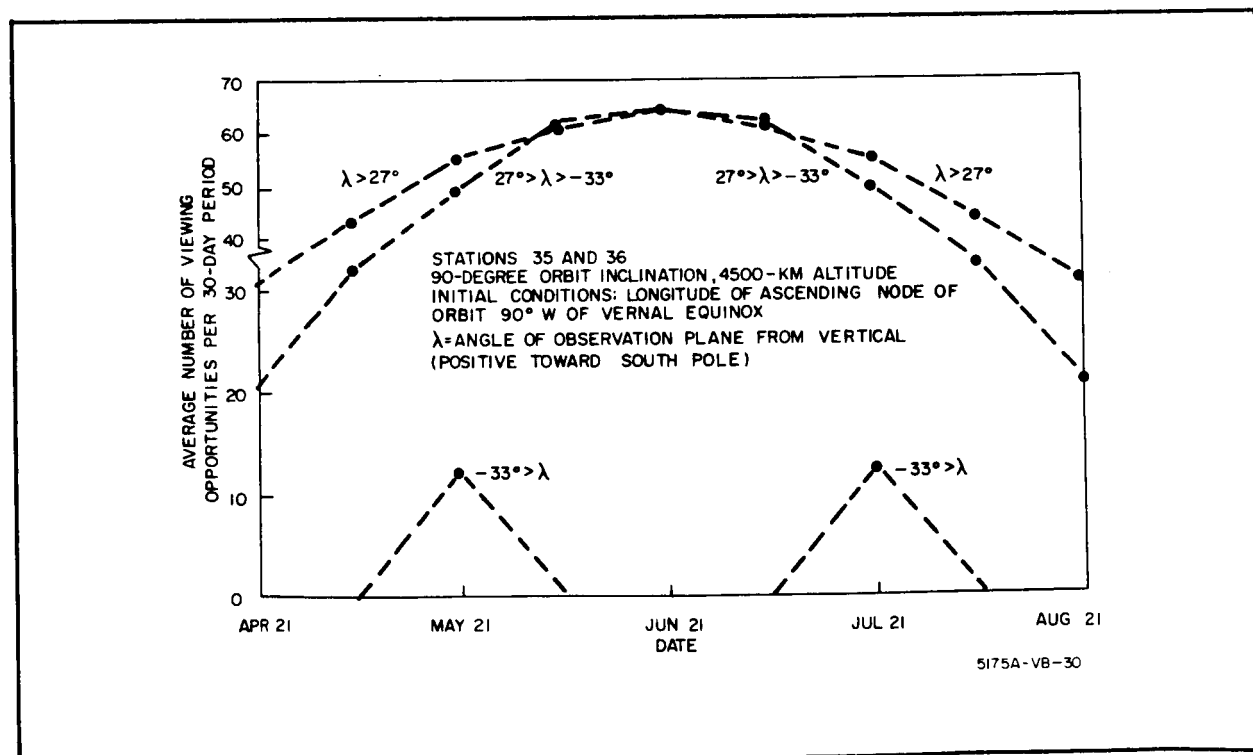
Figure 42. Observation Opportunities for Stations 35 and 36
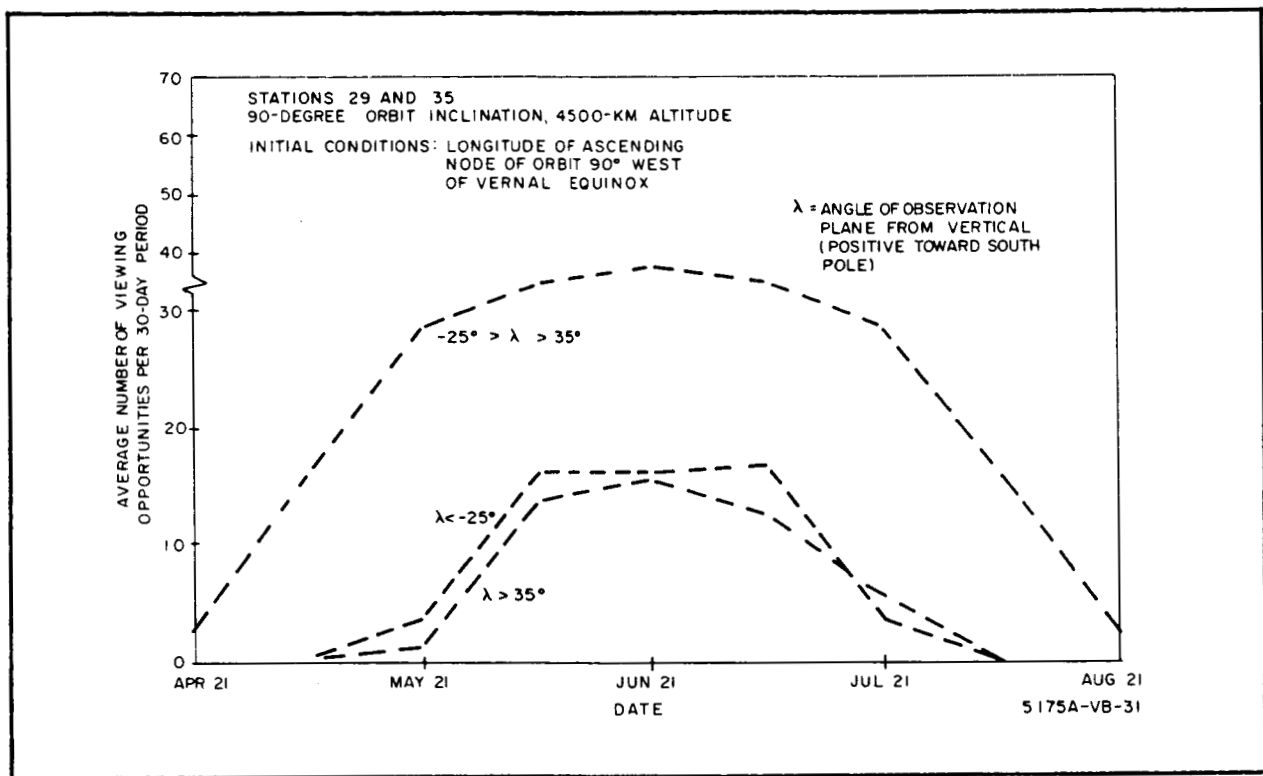
Figure 43. Observation Opportunities for Stations 29 and 35

a spring launch date will provide the sunlit satellite conditions required for satellite inflation.

e. Although suitable orbits have been found with respect to viewing opportunities at the south pole, suitability of these orbits for observation at the north pole stations is still not resolved. It appears that no difficulty would be encountered with the 90-degree orbit. Because of the 6-month delay between darkness at the south pole and darkness at the north pole, orbit perturbations other than regression of the nodes must be considered to obtain valid results with respect to the suitability of an orbit for observations at both poles. As a result of the increased complexity of this problem, further graphical analysis does not appear warranted.

Conclusion. - From the result obtained above, the following conclusions are apparent:

a. Observations cannot be made from all three polar stations simultaneously, due to darkness limitations.

b. A 4000-km, 80-degree orbit appears of marginal value, giving no opportunity of obtaining large (i.e., 60-degree) angles of intersection between the observation planes of various observations.

c. With a 4000-km, 83-degree or 4500-km, 90-degree orbit, observations can be made from any pair of stations simultaneously, with the conditions of sunlit satellite, suitable satellite altitude, and station darkness being satisfactorily fulfilled.

d. Initial orbit-plane orientation (i.e., launch date and time) will be important in obtaining satisfactory observations at all polar station pairs.

Method of Analysis

Initially, the polar station positions were plotted on a globe and areas were determined corresponding to satellite mutual visibility at two or three stations. From examination of the relative size and location of these areas it appeared that the south pole stations were more critically located, and so further effort was confined to these stations.

The station positions were then plotted on polar graph paper (figure 44). Areas of visibility were plotted for circular orbits at altitudes of 4000 and 5000 km, such that if the suborbital point of a satellite falls within the area of visibility of any particular station, the line of sight from that station to the satellite will be within 60 degrees of zenith. The intersection of the areas of visibility of two or three stations will thus display the area of common visibility corresponding to those stations. In addition, curves of constant observation plane angle were plotted for the various baselines, for circular orbits at altitudes of 4000 and 5000 km. (The observation plane angle with respect to a particular baseline was defined as the angle between the plane containing the baseline and the satellite, and the plane containing the baseline and the earth's center, positive toward the south pole.)

A second graph was prepared showing the contours of darkness (sun 18 degrees below the horizon) and showing the earth's shadow on the orbital sphere (figure 45). Both of these were determined as a function of the sun's position
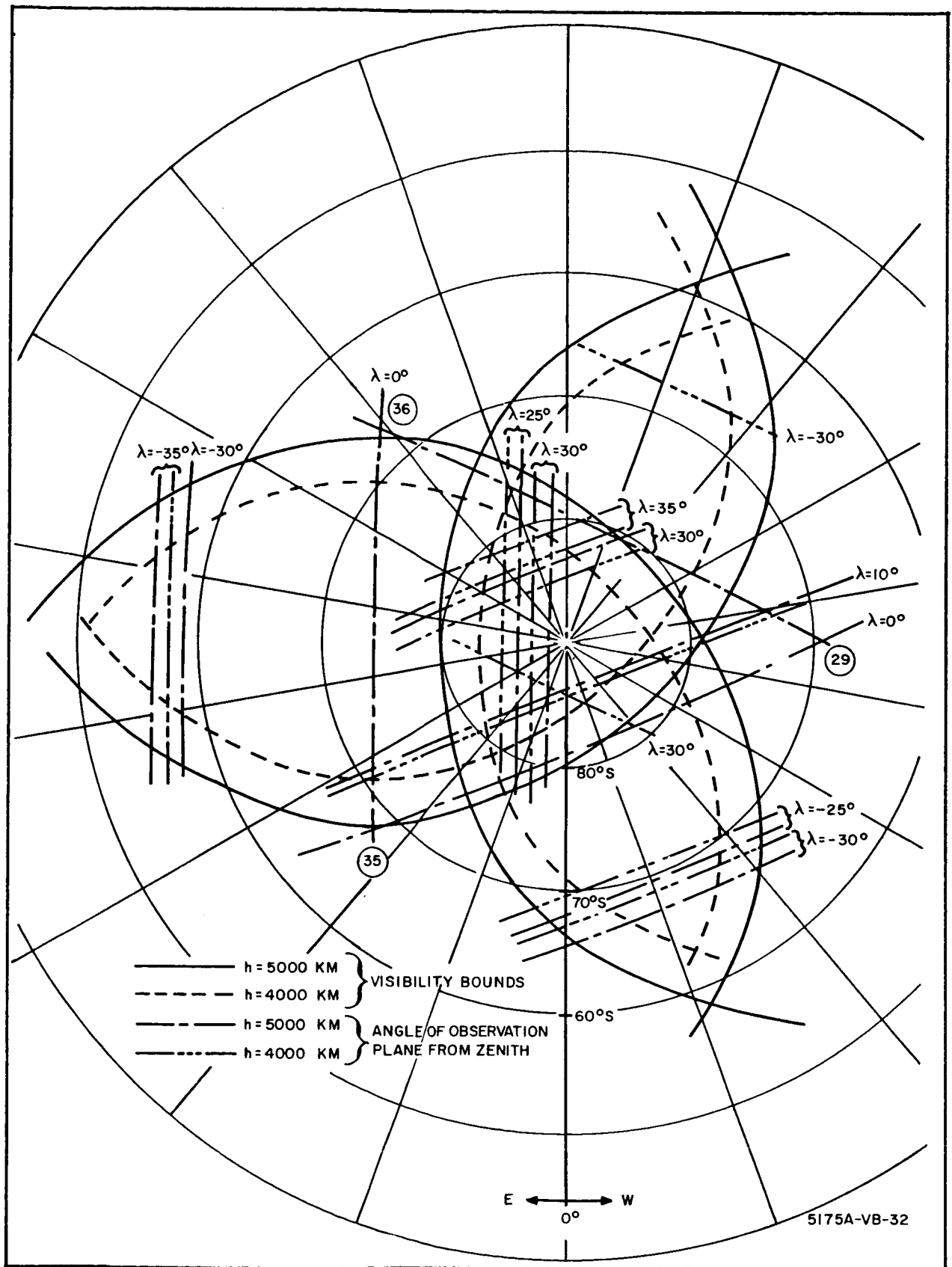
Figure 44.   Station Locations and Viewing Areas

APR 21, AUG 21
MAY 6 AUG 7   MAY 21, JUL 21
MAY 6, AUG 7   JUNE 21
MAY 21, JUL 21
JUNE 21

APR 21, AUG 21
MAY 6, AUG 7
MAY 21, JUL 21
JUNE 21

- - -   18° DARKNESS
———   EARTH SHADOW ON
      ORBIT SPHERE (4000 KM)
—·—   EARTH SHADOW ON ORBIT
      SPHERE (5000 KM)

DIRECTION
OF SUN

5175A-VB-33
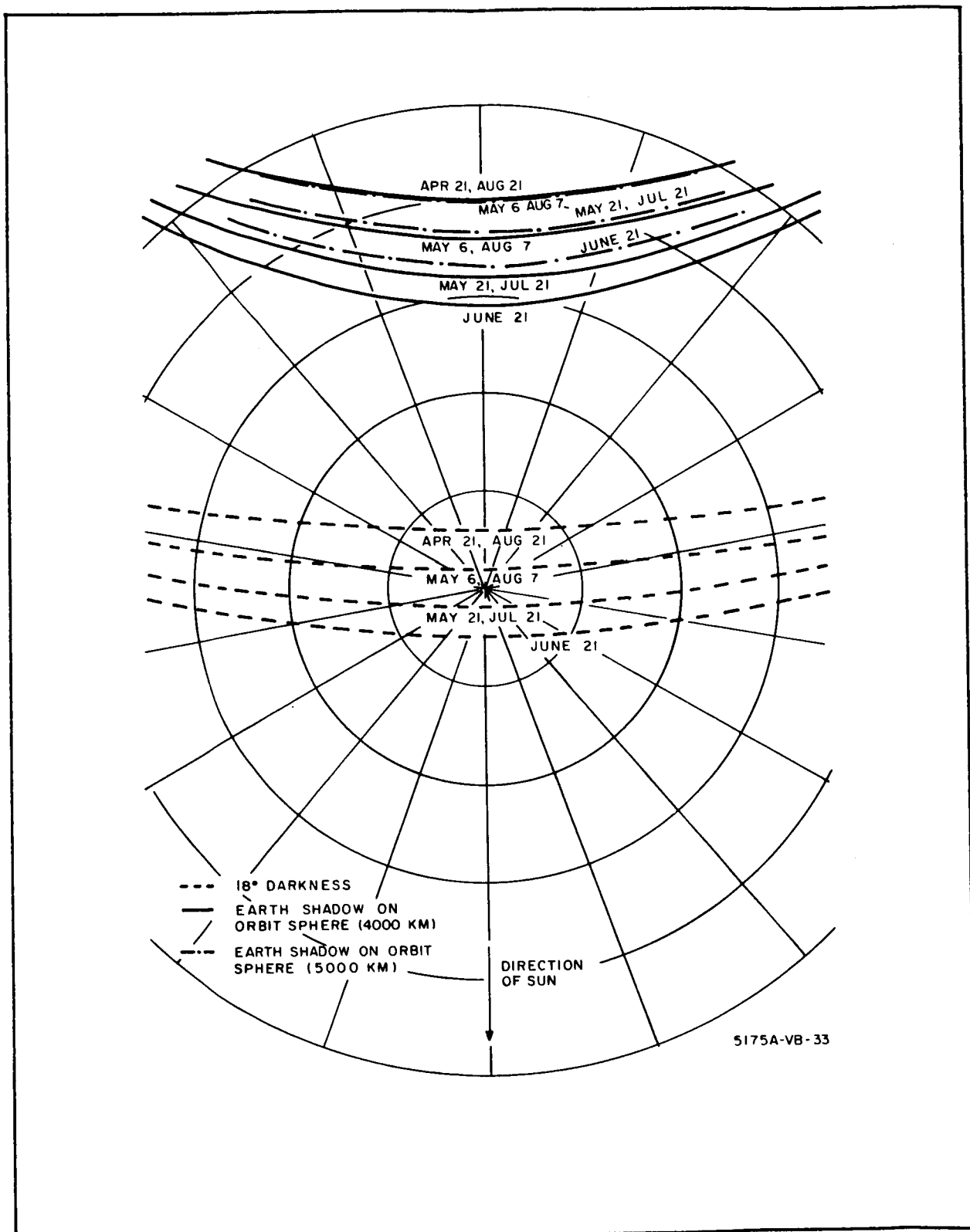
Figure 45.   Station Darkness, Earth Shadow

on the ecliptic. A third graph was prepared showing the suborbital trace of orbits at 80-, 83-, and 90-degree inclination (figure 46). By overlaying these graphs and rotating them with respect to each other, arcs of the orbit were determined which correspond to conditions of mutual visibility, station darkness, and sunlit satellite for various baselines and for various positions of the satellite orbit with respect to the sunline. This arc varies as the graph of area of visibility is rotated, corresponding to the 24-hour daily rotation of the earth. The average number of viewing opportunities in a 30-day period, assuming an asynchronous relation between the orbit period and the earth's period of rotation, was then determined by multiplying 30 by the ratio of the arc of earth's rotation during which satisfactory visibility conditions are maintained to the arc of earth's rotation corresponding to one orbit period. The arc of orbit corresponding to otherwise satisfactory visibility conditions was required to be at least of 2 minutes duration to be accepted.

Figure 47 illustrates the procedure described above on an assumed date of May 6. The regression of nodes of the orbit was factored into the solution by appropriately rotating the orbit graph with respect to the sunline; however, for simplicity, the orbits were considered to be circular, and other perturbations were ignored. With the ascending node 108 degrees west of the vernal equinox on April 21, it will be 111.5 degrees west by May 6. The sunline will be 45 degrees east of the vernal equinox on that date. These directions determine the relative orientation of the graph containing the earth's shadow and darkness contours and the graph containing the orbit trace. The rotational position of the earth may be specified by the position of the Greenwich meridian. Note that in the position shown (approximately 0140 Greenwich time), the orbit trace cuts the viewing area common to stations 29 and 35. The scale of degrees along the orbit trace represents the satellite angle measured from the ascending node, while λ is the angle of the observation plane from vertical (positive toward the south pole). In this instance the satellite is visible and sunlit, with the angle of the observation plane from vertical more than 26 degrees to the north, for a period of about 5 degrees, which exceeds a duration

ASCENDING NODE

−60° −70° −80° −100° −110° −120°

−60° −70° −80° −100° −110° −120°

80°

70°

60°

4000 − KM ORBIT : 4° = 2 MINUTES
5000 − KM ORBIT : 3.5° = 2 MINUTES

90°

5175A-VB-34
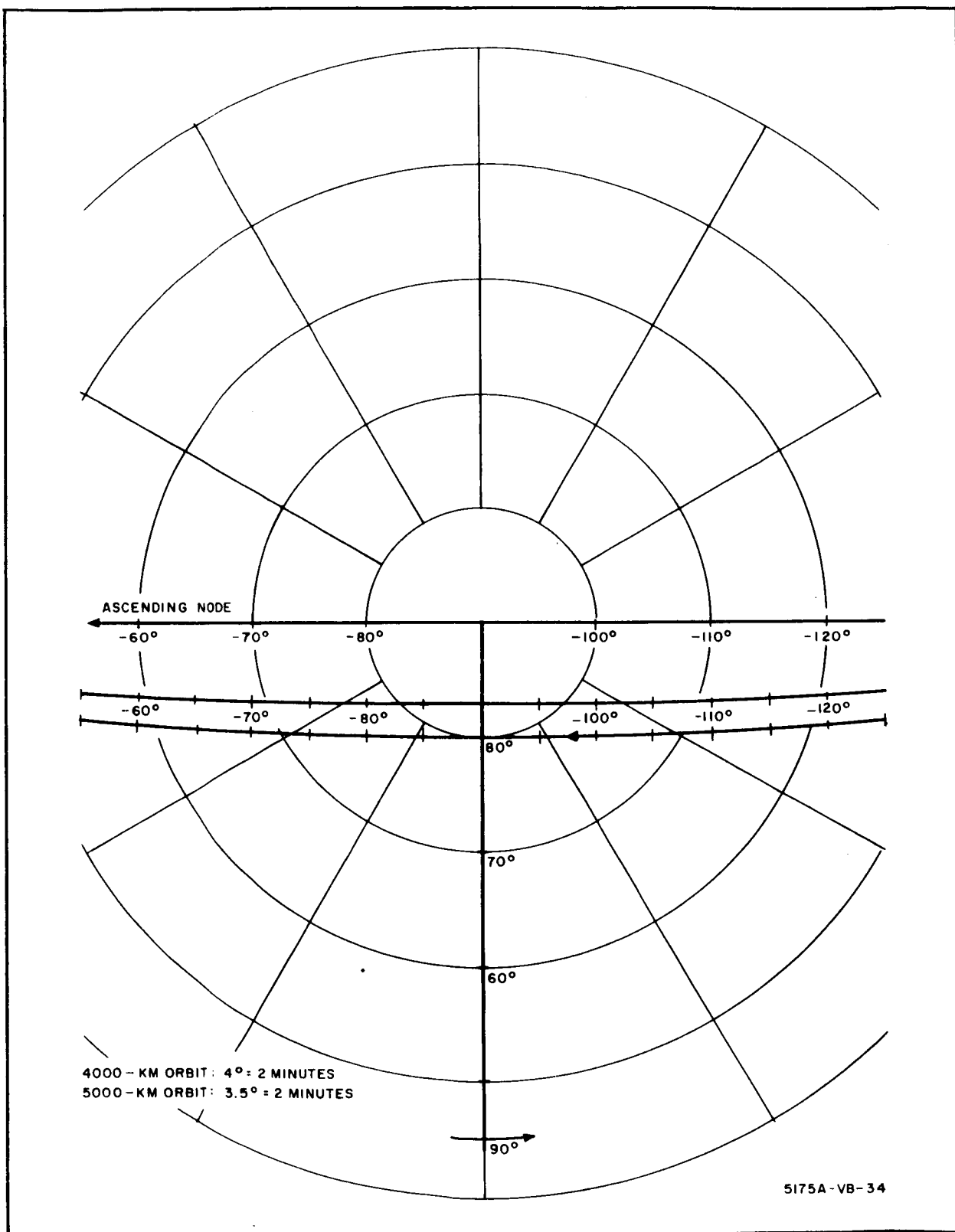
Figure 46. Orbit Traces
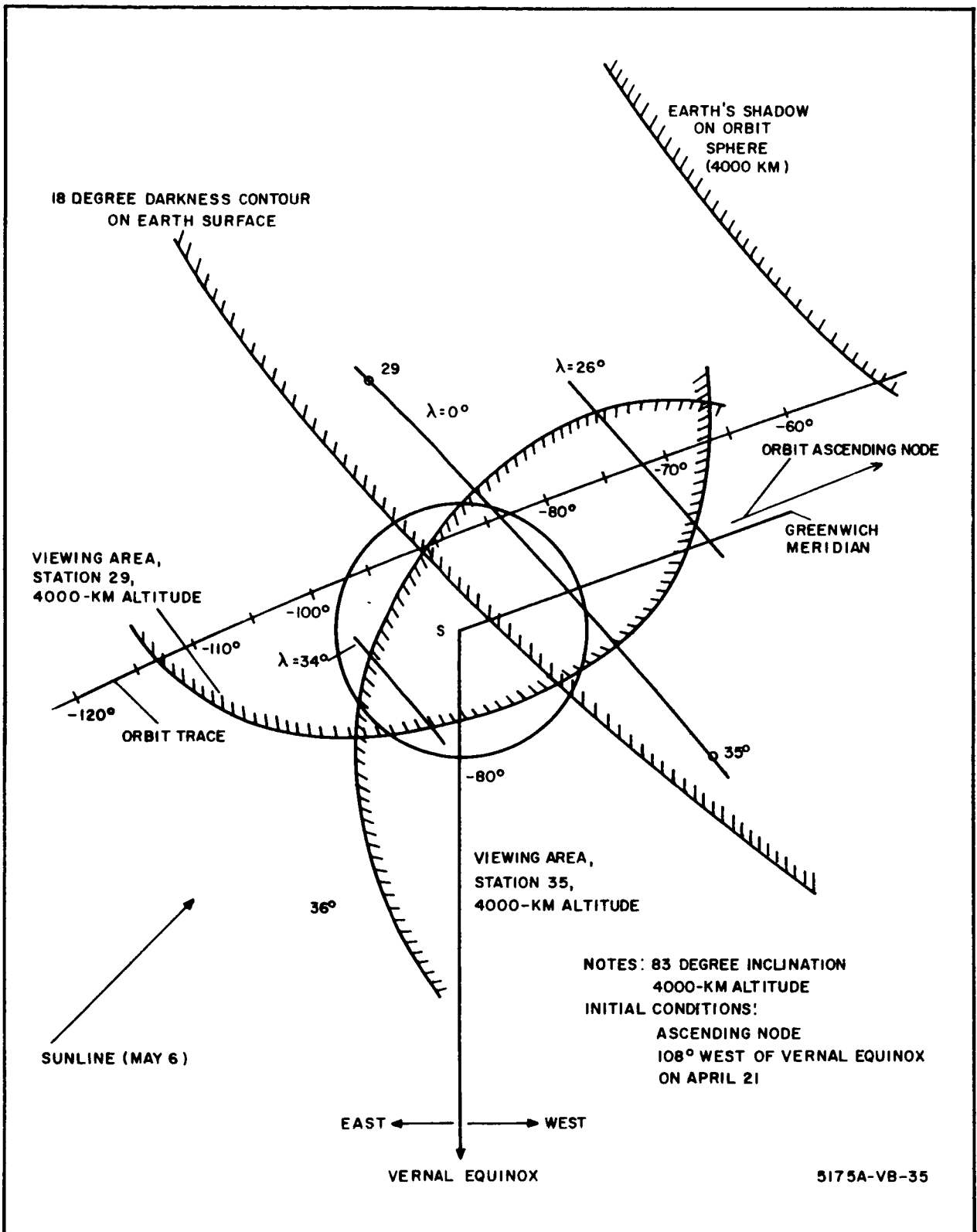
Figure 47. Illustration of Arc Computation

of 2 minutes. Stations 29 and 35 are also simultaneously in darkness. The arc of the orbit for which λ > -26 degrees is about 18 degrees. If the graph showing the visibility areas were rotated with respect to the other graphs, it would be found that the arc of orbit satisfying the various visibility conditions is greater than 2 minutes (λ < -26 degrees) whenever the Greenwich meridian is between 116 and 99 degrees west of the vernal equinox, for a total arc of earth rotation of 17 degrees. Since the arc of earth's rotation corresponding to one orbit period is about 45 degrees, the average number of viewing opportunities per 30-day period with λ < -26 degrees is about $(\frac{17}{45})$ 30 = 11.3.

Derivation of Formulas

<u>Viewing Area.</u> - The boundary of the viewing area corresponding to any given station is the intersection of a cone (having its vertex at the station and its axis along the station vertical) with the orbit sphere. Circular orbits have been assumed in this case. The angle which any ray in the cone makes with station vertical is β = 60 degrees. Referring to figure 48, part a, XYZ is a coordinate system with X along the vernal equinox and Z toward the south pole. $_1x$, $_1y$, $_1z$, and $_2x$, $_2y$, $_2z$ are aligned with $_1z$ and $_2z$ coincident with Z, and with $_1x$ in the plane of the Greenwich meridian and $_2x$ in the plane of the station meridian. From spherical trigonometry,

$$\cos a = \cos b \cos c + \sin b \sin c \cos A; \quad b = 90 + \phi \ (\phi < 0)$$

$$\frac{\sin C}{\sin c} = \frac{\sin A}{\sin a}; \quad \sin C = \sin A \frac{\sin c}{\sin a}$$

Referring to figure 48, part b,

$$\frac{R_e}{\sin a} = \frac{R_e + h}{\sin (180 - \beta)}; \quad a = \sin^{-1}\left[\frac{R_e}{R_e + h} \sin (180 - \beta)\right]$$

$$c = \beta - a$$

The longitude and latitude of the projection upon the earth's surface of points on the intersection of the viewing cone with the orbit sphere are then given by

$$\phi_v = -90 + a$$

$$\lambda_v = \lambda \pm C$$
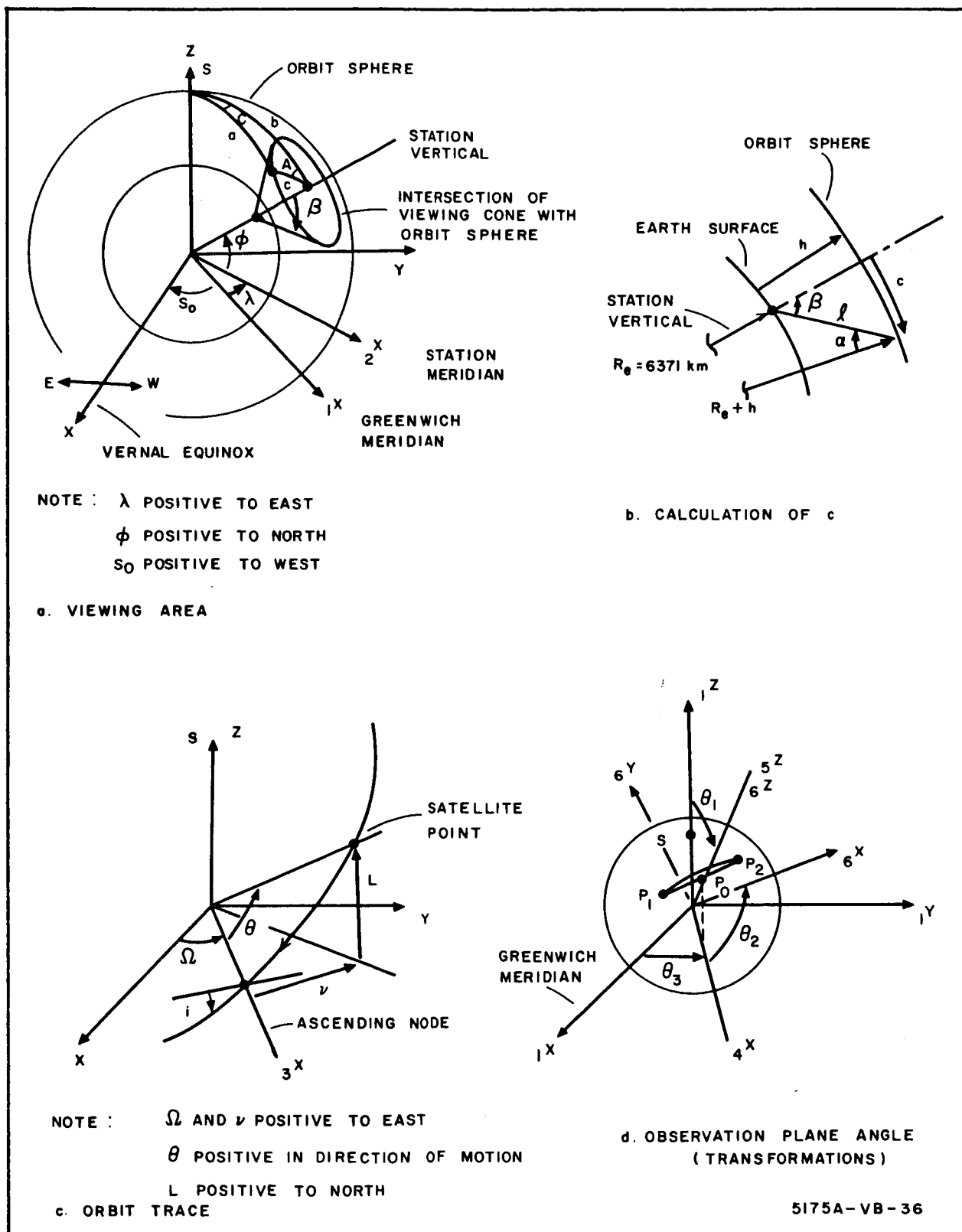
Figure 48. Derivation of Formulas

<u>Satellite Orbit.</u> - Referring to figure 48, part c, $_3x$, $_3y$, $_3z$ are axes aligned with $_3z$ along Z and with $_3x$ along the ascending node of the orbit. Ω is the longitude of the ascending node, measured east from the vernal equinox. The equation for the orbital plane in $_3x$, $_3y$, $_3z$ axes is:

$$(-\sin i)\,(_3y) + \cos i\,(_3z) = 0$$

For given θ, the satellite position in the $_3x$, $_3y$, $_3z$ axes is given by:

$$_3x_1 = \cos\theta\,(R_e + h)$$

$$_3y_1 = -\sin\theta\,\cos i\,(R_e + h)$$

$$_3z_1 = -\sin\theta\,\sin i\,(R_e + h)$$

L and ν are then given by

$$\sin L = \sin\theta\,\sin i$$

$$\tan \nu = \tan\theta\,\cos i$$

Traces of satellite orbits were plotted in figure 46 with θ as a parameter for inclinations of 80, 83, and 90 degrees. For orbit altitudes of 4000 and 5000 km, an arc of 2 minutes duration corresponds to an arc length of 4 and 3.5 degrees respectively.

<u>Angle of Observation Plane.</u> - The angle of the observation plane associated with a satellite position and a pair of stations may be defined as the angle between: a plane through the satellite position and the stations, and the plane through the earth's center and the stations. For the station pairs near the south pole, this angle was taken as being positive toward the south pole. The locus of satellite positions at a given observation plane angle was obtained by projecting upon the earth's surface the intersection with the orbit sphere of rays lying in the observation plane.

Referring to figure 48, part d, suppose the two stations of the station pair are P_1 and P_2, with P_0 the midpoint of the straight line joining them. ($_4x$, $_4y$, $_4z$) are axes obtained by rotating in a positive (right-handed) sense with respect to ($_1x$, $_1y$, $_1z$) axes through θ_3 about $_1z$, so that P_0 is in the ($_4x$, $_4z$) plane. Thus, if the longitude and latitude of coordinates of P_1 and P_2 are

(λ_1, ϕ_1) and (λ_2, ϕ_2), the coordinates of P_1 and P_2 in $(_1x, _1y, _1z)$ are:

$$\begin{bmatrix} _1x_1 \\ _1y_1 \\ _1z_1 \end{bmatrix} = \begin{bmatrix} R_e \cos\phi_1 \cos\lambda_1 \\ -R_e \cos\phi_1 \sin\lambda_1 \\ -R_e \sin\phi_1 \end{bmatrix} \quad ; \quad \begin{bmatrix} _1x_2 \\ _1y_2 \\ _1z_2 \end{bmatrix} = \begin{bmatrix} R_e \cos\phi_2 \cos\lambda_2 \\ -R_e \cos\phi_2 \sin\lambda_2 \\ -R_e \sin\phi_2 \end{bmatrix}$$

The coordinates of P_0 are:

$$P_0 = \begin{bmatrix} _1x_0 \\ _1y_0 \\ _1z_0 \end{bmatrix} = \begin{bmatrix} 1/2\,(_1x_1 + _1x_2) \\ 1/2\,(_1y_1 + _1y_2) \\ 1/2\,(_1z_1 + _1z_2) \end{bmatrix}$$

and the angle θ_3 is given by:

$$\tan\theta_3 = \frac{_1y_0}{_1x_0}$$

The transformation from $(_1x, _1y, _1z)$ axes to $(_4x, _4y, _4z)$ axes is:

$$\begin{bmatrix} _4x \\ _4y \\ _4z \end{bmatrix} = T_3 \begin{bmatrix} _1x \\ _1y \\ _1z \end{bmatrix} \quad ; \quad T_3 = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Let $(_5x, _5y, _5z)$ be axes obtained by rotating $(_4x, _4y, _4z)$ through an angle θ_1 about the $_4y$ axis, in a positive (right-handed) sense, until P_0 lies on the $_5z$ axis. Then

$$\tan\theta_1 = \frac{_4x_0}{_4z_0}$$

where

$$\begin{bmatrix} _4x_0 \\ _4y_0 \\ _4z_0 \end{bmatrix} = T_3 \begin{bmatrix} _1x_0 \\ _1y_0 \\ _1z_0 \end{bmatrix}$$

The transformation from $(_4x,\ _4y,\ _4z)$ to $(_5x,\ _5y,\ _5z)$ is:

$$\begin{bmatrix} _5x \\ _5y \\ _5z \end{bmatrix} = T_1 \begin{bmatrix} _4x \\ _4y \\ _4z \end{bmatrix} ; \quad T_1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 \\ 0 & 1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 \end{bmatrix}$$

Finally, let $(_6x,\ _6y,\ _6z)$ be obtained by rotating $(_5x,\ _5y,\ _5z)$ about $_5z$ in a positive (right-handed) sense through the angle θ_2, until P_2 lies on the $_6x$ axis. Then:

$$\tan\theta_2 = \frac{_5y_2}{_5x_2}$$

where

$$\begin{bmatrix} _5x_2 \\ _5y_2 \\ _5z_2 \end{bmatrix} = T_1\ T_3 \begin{bmatrix} _1x_2 \\ _1y_2 \\ _1z_2 \end{bmatrix}$$

and the transformation from $(_5x,\ _5y,\ _5z)$ to $(_6x,\ _6y,\ _6z)$ is:

$$\begin{bmatrix} _6x \\ _6y \\ _6z \end{bmatrix} = T_2 \begin{bmatrix} _5x \\ _5y \\ _5z \end{bmatrix} ; \quad T_2 = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 \\ -\sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Referring to figure 49, part a, if an observation plane is passed through P_1, P_0, P_2 at an angle a_ν from the $_6z$ axis (a_ν positive toward the $_6y$ axis), it will intercept the orbit sphere in a circle whose center is taken as the point P_3, and whose radius is r_3. If the coordinates of P_0 in $(_6x,\ _6y,\ _6z)$ are $(0,\ 0,\ _6z_0)$, then the coordinates of P_3 may be found geometrically to be:

$$P_3 = \begin{bmatrix} _6x_3 \\ _6y_3 \\ _6z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -_6z_0 \sin a_\nu \cos a_\nu \\ _6z_0 \sin^2 a_\nu \end{bmatrix}$$

a. OBSERVATION PLANE ANGLE

b. SHADOW ON ORBIT SPHERE

NOTE: ϕ_S POSITIVE TO NORTH

c. BOUNDARY OF REGION OF 18 DEGREE DARKNESS

5175A-VB-37

Figure 49. Derivation of Formulas

The coordinates of the endpoint of $\bar{r}_3$ are then:

$$\begin{bmatrix} {}_6 x_v \\ {}_6 y_v \\ {}_6 z_v \end{bmatrix} = \begin{bmatrix} r_3 \sin \theta_v \\ {}_6 y_3 + r_3 \cos \theta_v \sin a_v \\ {}_6 z_3 + r_3 \cos \theta_v \cos a_v \end{bmatrix}$$

These coordinates can then be transformed to $({}_1 x, {}_1 y, {}_1 z)$ axes:

$$\begin{bmatrix} {}_1 x_v \\ {}_1 y_v \\ {}_1 z_v \end{bmatrix} = T_3^{-1} T_1^{-1} T_2^{-1} \begin{bmatrix} {}_6 x_v \\ {}_6 y_v \\ {}_6 z_v \end{bmatrix}$$

The longitude and latitude of the projection of these points on the earth's surface can then be obtained from:

$$\tan (-\lambda) = \frac{{}_1 y_v}{{}_1 x_v}$$

$$\tan (-\phi) = \frac{{}_1 z_v}{{}_1 x_v} \cos (-\lambda)$$

<u>Earth Shadow on Orbit Sphere.</u> - Referring to figure 49, part b, $({}_7 x, {}_7 y, {}_7 z)$ are axes aligned with ${}_7 z$ south and with the sun in the $({}_7 x, {}_7 z)$ plane at an angle ϕ_5 from the ${}_7 x$ axis (positive to north). $({}_8 x, {}_8 y, {}_8 z)$ are axes obtained by rotating $({}_7 x, {}_7 y, {}_7 z)$ through ϕ_s about ${}_7 y$, so that the corresponding transformation is:

$$\begin{bmatrix} {}_8 x \\ {}_8 y \\ {}_8 z \end{bmatrix} = T_s \begin{bmatrix} {}_7 x \\ {}_7 y \\ {}_7 z \end{bmatrix} ; \quad T_s = \begin{bmatrix} \cos \phi_s & 0 & -\sin \phi_s \\ 0 & 1 & 0 \\ \sin \phi_s & 0 & \cos \phi_s \end{bmatrix}$$

The coordinates of points of intersection of the earth's shadow on the orbit sphere, in $({}_8 x, {}_8 y, {}_8 z)$ axes, are:

$$\begin{bmatrix} _8x_s \\ _8y_s \\ _8z_s \end{bmatrix} = \begin{bmatrix} -(R_e + h)\cos\theta_5 \\ (R_e + h)\sin\theta_5 \sin\theta_4 \\ (R_e + h)\sin\theta_5 \cos\theta_4 \end{bmatrix}$$

where

$$\theta_5 = \sin^{-1}\frac{R_e}{R_e + h}$$

and θ_4 is a variable parameter. Transforming to $(_7x, _7y, _7z)$ axes:

$$\begin{bmatrix} _7x_s \\ _7y_s \\ _7z_s \end{bmatrix} = T_s \begin{bmatrix} _8x_s \\ _8y_s \\ _8z_s \end{bmatrix}$$

Finally, angles ϕ_3 and ϕ_4 (from which the projection of the earth's shadow on the earth's surface can be plotted in polar coordinates) can be found from:

$$\cos\phi_4 = \frac{_7z_s}{R_e + h}$$

$$\tan\phi_3 = \frac{_7y_s}{_7x_s}$$

Region of Earth's Surface in 18-Degree Darkness. - The boundary of the region earth's surface in 18-degree darkness can be found in a manner similar to that used in finding the earth's shadow on the orbit sphere. Referring to figure 49, part c, the coordinates of the boundary in $(_8x, _8y, _8z)$ axes are:

$$\begin{bmatrix} _8x_d \\ _8y_d \\ _8z_d \end{bmatrix} = \begin{bmatrix} -R_e \cos\theta_7 \\ R_e \sin\theta_7 \sin\theta_6 \\ R_e \sin\theta_7 \cos\theta_6 \end{bmatrix} \qquad (\theta_7 = 90 - 18 = 72 \text{ degrees})$$

where θ_6 is a variable parameter.

Transforming to $(_7x, \ _7y, \ _7z)$ axes:

$$\begin{bmatrix} _7{}^x d \\ _7{}^y d \\ _7{}^z d \end{bmatrix} = T_s^{-1} \begin{bmatrix} _8{}^x d \\ _8{}^y d \\ _8{}^z d \end{bmatrix}$$

The angles ϕ_5 and ϕ_6 (from which the boundary can be plotted in polar coordinates) can then be found from:

$$\cos \phi_6 = \frac{_7{}^x d}{R_e}$$

$$\tan \phi_5 = \frac{_7{}^y d}{_7{}^x d}$$

If $_7{}^x d \approx R_e$, then the following formula can be used to obtain ϕ_6:

$$\sin \phi_6 = \frac{\sqrt{_7{}^x d^2 + _7{}^y d^2}}{R_e}$$

IX-24

APPENDIX X

COMPLETE 36-STATION, 5-YEAR RUN

The following tables show the results obtained from the complete 5-year run using the orbit characteristics of orbit 1. These characteristics were:

Altitude - 4250 km (circular)

Inclination - 87 degrees

Right Ascension of the Ascending Node - 345 degrees

Launch Date - 1 June 1966

(Launch time approximately 6:30 a.m. Pacific Standard Time if launched from Pacific Missile Range.)

The meanings of the various entries are explained in the section entitled "Discussion and Results" (Final Orbit Selection).

BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
1-2	G					8	34	23	13	13								3	17
	M						2	1										1	
	+						9	3											
	−						9	11											
1-3	G					17	33	25	20	20	2								10
	M						1	2	1										
	+						1												
	−					12	30	21	9	7	1								10
1-4	G					6	31	16	2	1								10	20
	M						2	7	2										
	+																		
	−						7	9											20
1-5	G						1	9	2										
	M																		
	+																		
	−																		
1-6	G					12	42	39	18	6								12	38
	M							1											
	+																		
	−					11	26	4											1
1-7	G					1	13	21	4	5								6	18
	M						8	5	2	1								4	1
	+						5	2										8	
	−						7	2											
2-3	G			1	7	2									5				
	M														1				
	+				2														
	−			1	7	3									5				
2-7	G		1	2	1				3	2	8								1
	M			2	1						5								
	+										1								
	−		1																
2-8	G	1	11	1	1	4	7	2			2	1		5	2	8	1		
	M		3	2			4	2						5	1	1			
	+		9					1						4	3				
	−					1	7				3	2					2		
2-9	G			4	1	7	8	6			1	9			3	3	13		4
	M			1			5				3	4			1	4	2		9
	+					3		2									4		
	−					4	6					12				2			
2-10	G			1		4		1			1	5				10	4		7
	M			1								1				2			2
	+																		
	−					1		1				4				8	3		
3-4	G				2	1		2	11	5								2	11
	M						1	1											1
	+																		
	−				2	2		2	4	1									6

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BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
3-10	G	2	7		8	8	15	12			9	3			11	21		2	3
	M	2	3		1		3	1				1						4	
	+						3							2	4	2			
	−				3	2	10	3			1	5				4			
3-11	G		1	1	5	14	6	6			11	4			4	10			7
	M		2				2	1			1	1							1
	+							3		1									
	−				1	9	8				4	4				8			
4-5	G					5	12	23	32	13								19	30
	M					4	18	20	14									6	15
	+				1			1	1		1								
	−						20	30	26	8								33	41
4-11	G		4	9	3	15	6	6		2	2					2	3	3	6
	M		3	2			3	2		2									2
	+			6			1	3											1
	−					8	6									1	4		
4-12	G			2	12	3	22	13			16	3							10
	M			1	1		5	2		3	1	2							2
	+						4	4								5			4
	−					2	18				8	6							
4-13	G				6	25	8				5	3							
	M					1	3												
	+					5													
	−				7	19	10				6	4							
5-6	G				8	21	48	50	32	8								9	46
	M				7	1													
	+																		
	−				3	28	81	84	49	9								13	64
5-13	G				4	3	15	11	2	4	3							6	16
	M						4	2	1	5								5	7
	+						3	5											
	−				1	2	10											12	5
5-14	G				4	6	19	15			8								13
	M					1	5	3		2									3
	+						2	2											2
	−					11		4											
5-15	G				1	12	11	2		2	7	2							
	M					5		5			4	4							
	+																		
	−				1	4	13	4		2	6	3							
6-7	G				6	21	14	3		4	6								1
	M					2	5			3	3								6
	+					4													
	−				7	19	20			2	4								2
6-15	G				2	1	9	9		1								4	16
	M						6	1	1	3								4	10
	+						13	3										8	
	−						1												

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END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965

BASELINES		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
6-16	G				4	20	24	24	3	10	10	6						7	18
	M				2		2	2	3	3		2						1	1
	+					5	9	2										9	
	−					4	15	10			6	10							
7-8	G			4	6	12	2	5			4	7			4	11	7		5
	M						1	1				5				2			
	+					4		2											
	−					2	3					3				7	6		
7-16	G			2	1	15	1	2		2						1			1
	M							2											
	+			2		10										1			1
	−			1		8	1												
7-17	G	3	15	11	3		5	8			8	1		11	8	8		1	1
	M		2	1			2	2						5					
	+		11	4				3						6		2			
	−						6	1			4	2			2	2			
8-9	G				3		1	3			12			4			10		
	M										1								
	+							2			9			2			9		
	−						1	1			10			4			10		
8-17	G			5	3	6					4				1	1	4		
	M					1											1		
	+					2											2		
	−					4					3								
8-18	G	2	3			9	1		3	2	2			3			9		
	M		1				1			3	1			1					
	+	2	1			1													
	−					6	2		1	4	2						7		
8-19	G	3						5	5		12	1	2				3	3	6
	M	3					1	2	1	2	5		2						
	+	3				1		5			2								
	−										2							6	
9-10	G	1	1	2	1						9			10	1	1			
	M										2				2				
	+										8			1					
	−	1	1	3							6			7		2			
9-19	G	10			1	6	4		1	5	1		5	5			8	1	
	M					1	2			2			2				1	3	
	+	3											4						
	−					6	5		1	4							10	2	
9-20	G				1			3	5	3							1	1	1
	M							4	1	6									
	+				1					2									
	−																2	1	
10-11	G			1	3	3									3				
	M					2													
	+																		
	−					1									1				

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BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1,1965																	
		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
10-20	G	8	2			7	4	3	1		2		5	6			7	7	5
	M	3					2		2		5		2						6
	+							2			2								
	−	6				6	6				2							1	
10-21	G		1			2		6	3		3	5		2		1	7		
	M		1					1	1					1		2			
	+							4											
	−					2					1	7					5		
11-12	G				1	12						2			10	7			
	M											1							
	+					12										4			
	−					12									8	7			
11-21	G	7	12	2		5	8	3	3		12	1	3	10			4	1	3
	M	3	1	1		1	1	1	2				3	3				1	7
	+							2						2					
	−	6	6			2	8	2			9							2	
11-22	G					7	1	7	2		13	10				2	8		4
	M					1		2			8					2	1		
	+							4											
	−					5	2					12				1	8		
12-13	G		3	1		9	1							4	2	12			
	M			1		1													
	+					9										6			
	−		3			1								3	2	10			
12-22	G	8	12	1		7	7	2	1		4		1	11		1	3	1	2
	M	2					1		3		1		2	3					4
	+							1			1			2					
	−	9	1			5	9				2						2		
12-23	G					5	6	3			5	3		2	1	3	8		3
	M						1	1						2		1			3
	+					1	6										3		
	−					1					1	4				5	2		1
13-14	G			4	3	11	1				1					1			
	M						1				1								
	+					8	1				1								
	−			3	2	11	1								1	1			
13-23	G	4	10	2		18	9	4	3		6			9	1	3	13	1	6
	M	3	1	1			5	1			1			4	2	3	2	1	
	+	3				5													
	−					1	11		2		6						6	2	
13-24	G			1		13	2	4			1	6				7	9		3
	M			1								2			3	2	1		
	+			2															
	−					8	3					4				5	8		
14-15	G		1	1		15	4			3				1		3			
	M																		
	+					12	3									3			
	−			1		12	2							1		3			

BASELINES		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
14-24	G	6	7	2			3	4	5		6	5	1	14	5	6			5
	M	1	1			1	1	1					2	1	1				4
	+	9	6					3			1		2	6	1				2
	−						4				3	4							
14-25	G			1	1	7	1	4	2		1	12			1	7	2		6
	M								1										
	+			2		4		4			1	1				1			6
	−					1	1					11				9	2		
15-16	G			2	1	17	2	1	1	2	2								
	M					11	1		1	7	1								
	+			2		16					1								
	−				1		2		1	2									
15-25	G	1	9	2			3	4			6			9	5	4		3	
	M	1	1				3						1	2					
	+	2	5					1			2			3					
	−						4				4				1	2			
15-26	G			4	3	24	2	4				8			1	7	6		2
	M							1				3							
	+					3		2							1	1			3
	−					10	3					4				7	3		
16-17	G			3	3	5	2	4	1		1	8			3	6			5
	M			1				1				1							
	+					2		5						1					1
	−					3	3					8				6			
16-26	G	4	18	6	2	1	3	7			15	2	2	17	8	6		3	
	M		2				1						2	2					
	+	5	11	4	2			4			2		3	10					
	−					1	5	1			10	3		2					
17-18	G	2	4	1		12		10	15	3	3		1	6			12		8
	M							1	3				1						2
	+	2						5	5				1	1					1
	−					9		1	1		5			1			10		
17-26	G			3	2	7					3			2		2	5		
	M		1								1	1				1	4		
	+		1			4					1			1	1	2			
	−					5					3								
17-27	G	5	5			13			3	2	16	9			2		8		
	M	2					1				2	1					2		
	+	4	1			3	2				1				7				
	−					4	2				9	8					5		
18-19	G	2	1	3	1			2	5	10			2	1					
	M																		
	+	2	1		1			1	5	8				1					
	−																		
18-27	G			2	2					4	13	2							
	M																		
	+										3								
	−				1					3	12	2							

END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965

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Baselines		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
18-28	G 18	4	2		9	3				5			11	4		2	17		
	M 6		4			1							5			2	5		
	+ 11												3						
	− 3									3							5		
19-20	G		2					2		1									
	M		2	1						1						1			
	+								2										
	−								2										
19-28	G 2				5	10							2				7		
	M 2			1		3							1	1			2		
	+ 1					1							2						
	− 1				4	14											4		
19-29	G 19		3	3									12	3		4			
	M 4		2									3	6	2					
	+ 6												4						
	− 6		1																
20-21	G				2	2				1							1		
	M				1	1											1		
	+				3					1									
	−					1											1		
20-29	G 21		6	7	3						16	34	14	1			7		
	M 7			1	3						3	8	11	1		3	7		
	+ 3																		
	− 2										12	16	7						
20-30	G		2					2	1							7			
	M		3					2	2										
	+															1			
	−																		
21-22	G				2				1	1									
	M								1										
	+																		
21-30	G 19		6		6	2		4	8	8			11	4			7	1	2
	M 3				3			2					6						
	+ 14								3				7						
	− 8				1	3		5		4								2	
22-23	G				1				1	13									
	M					1													
	+								1	9									
	−									6									
22-30	G 1					10	1			6							7		
	M				1					1							1		
	+									1									
	−					12				3							5		
22-31	G 10	1			2				12	8			6	2			2		
	M 3				1			2		2			2						
	+ 4								3				3						
	− 4				1														

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BASELINES		550	560	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
										END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965									
23-24	G										9			1					
	M				1						7			1					
	+										4								
	−																		
23-31	G	8	1			12	2			7	31	17	3	2			8		
	M	2				4				4	7	5	4	1			4		
	+	3									12								
	−					6				1	16	17	4				1		
23-32	G	1	1					6	17	6	1				2				
	M	1						3	1	1			1						
	+	1						5	9										
	−								3		2								
24-25	G					7	1				2			2			1		
	M						1												
	+					6					1			1			1		
	−					4					2			1					
24-32	G	9	2		1	14	3			12	1		8				14	4	3
	M				1	1	1		1	4			2				2	4	
	+												6						
	−	7	1			7	6			15							10	8	
24-33	G							2	4	11	29	16							
	M							4	2	4	6	1		1					
	+								1	4	2								
	−										5	11							
25-26	G			5	2	7					3	2		2	1	2	2		
	M					1											1		
	+					5					3	1							
	−					5					1	1		1			1		
25-33	G	5	1			6	3		1	6	31	25	5	2			1	6	
	M	2				1				10	3	4						3	
	+	4									4	2							
	−					3					8	11						6	
25-34	G		1			10		6	8	5	15	3		1			9		1
	M							2	4	3									4
	+							4		5	7								
	−					7					9	4					7		
26-27	G	1				10	1	9	8	4	1		2	2			11		6
	M							3	2	5	1		1						4
	+							6		1									
	−					8			1								7		
26-34	G	5	2			10	7	1		4	3		3	2			9	6	2
	M	8					1	2		8	2		1	3			1	1	
	+										3		3	3					
	−					3	9			4							1	9	
27-28	G	5	1		1					2	9	2	4	2		4	1		
	M	1	1							1	2		3						
	+	3	1										1						
	−									2	1	2							

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BASELINES		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
27-34	G	1	2	1	1			1		3									
	M																		
	+			6	2														
	−	1	4	2	1		1			2									
27-35	G	2	1	1	5						18	30	5			6	3		
	M																		
	+				3						3	7	7				6		
	−	3			3						3	18	8						
28-29	G	34	12	2								28	44	30	28	16			
	M	1																	
	+	48	12	2								44	67	21	11	9			
	−													4		3			
28-35	G	21	11	4	7							7	39	9	12	21	7		
	M		1										1	1		3			
	+																		
	−	41	21	4	8							12	68	18	10	17	7		
29-30	G	8	4	5	8						1	9	9	15		11	3		
	M																		
	+	9											10	4	6	10	9		
	−	10	4		1							14	15						
29-35	G	3	1										2	1					
	M																		
	+																		
	−																		
29-36	G	42	19									6	24	39	27				
	M																		
	+														1				
	−													1	5				
30-31	G		1	4	1								1			9			
	M															2			
	+												2	1	4	7			
	−		1	4	1								1			4			
30-36	G	4	3	1									3						
	M																		
	+	10	2									1	13	5		2			
	−	3	1																
31-32	G			1						9	2								
	M																		
	+			1						6	2				1				
	−			1						5	1								
31-36	G	34	10	13	5							4	30	25	11	23			
	M													2	20	4			
	+	26	1	2								2	32	7		8			
	−	7			1							8		2	1	2			
32-33	G	2	1							3						2			
	M																		
	+	2	1	2						3						1			
	−	1								3						2			

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BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
32-36	G	21		4	8	3							11	9		8	16		
	M	4											1	1		2	5		
	+	9											4						
	−	12				1							2	2			6		
33-34	G	1		2	9	2				4			1	1			3		
	M			3	6												1		
	+	1		2		1				4				1			3		
	−	1											1						
34-35	G	8	1	1	1						10	24	18						
	M	5		2	1						5	4	9						
	+	1																	
	−	3									6	14	10						
35-33	G	20	9	4	2								21	9		6			
	M	2										2	5	3		1			
	+	18											20						
	−	4	2	2	1														
35-36	G	40	13	1		38	1			15	78	45	54	48	48	11	57	4	
	M		1							1	1					4			
	+		2								2								
	−	39	11		1							11	44	46	45	6			
36-33	G	2			3						25	22	6			2	6		
	M	5			1						2	10	9	1					
	+										3								
	−	3			3						11	16	9						
	G																		
	M																		
	+																		
	−																		
	G																		
	M																		
	+																		
	−																		
	G																		
	M																		
	+																		
	−																		
	G																		
	M																		
	+																		
	−																		
	G																		
	M																		
	+																		
	−																		
	G																		
	M																		
	+																		
	−																		

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
1 2 3	G					14	4	15	13	14	8								11
	M						1	1	6	4	1								4
	+																		
	−					14	7	5	3	6	5								5
	+					3		12	4	7	4								7
	−																		
	+					10	3	4			3								3
	−					12	2	14	19	16	8								11
1 3 4	G					2			4	3									
	M								2	2									
	+																		
	−					4			8	6									
	+									1									
	−																		
	+																		
	−					1			4	3									
1 4 5	G								1										
	M																		
	+																		
	−																		
	+																		
	−																		
	+																		
	−																		
1 5 6	G						7	26	13										3
	M																		
	+																		
	−							3	7										
	+						7	22	4										3
	−																		
	+							11	6										3
	−						8	27	14										3
1 6 7	G					3	2	3	13	6									2
	M							3	13	7									9
	+																		
	−							3	13	6									2
	+							1	3										
	−																		
	+																		
	−								4	4									
1 7 2	G					5	2	5	4	6									4
	M					1	1	3	6	5									5
	+																		
	−					4	3	2	1	2									
	+							4	4	5									5
	−																		
	+					4	1	5	4	1									4
	−					1		3											2

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
2 7 8	G		1	13	15	9	1	3			2				7	4			4
	M			1	1	2	1	3			2				3	1			
	+			11	15	9		3							7	4			
	−			11	7	6		3			2				6	4			
	+			11	8	5		3							5	6			
	−																		
	+																		
	−			8	14	7	2	1			2				6				
2 8 9	G		8	13	1	14		4			10			2	11	17			
	M			2	5			3						4	6				
	+																		
	−		2	2		11		1			5				1	7			
	+		3	5		3					5		1		5	3			
	−																		
	+		5			11		3			7								
	−		8	22	2	14		4			10			2	17	22			
2 9 10	G		1	6	2	7		3			6				7	8	4		1
	M			2											2	2	3		
	+																		
	−			4		9					6				4	6	8		
	+		1	1		2		4			2				1	1	1		
	−					2											4		
	+					5					5						3		
	−		1	7		5		3			6				10	10			
3 2 10	G		1	14	19	12	3	3			8				8	1			4
	M			2	1		1	3							3				
	+			13	17	10	3	3			9				8				
	−			11	16	10	2	2			3				8	1			
	+			12	9	6		2			2				6				
	−																		
	+																		
	−			8	14	9	4	1			8				3	2			
3 10 11	G		1	8	3	2		1			4				3	7	1		
	M			1		1					1				1	1	1		
	+																		
	−			8	3	2		2			6				5	11	2		
	+		2	5		1		1			3				3	5			
	−										2					2	1		
	+					1										2			
	−		1	5	2	1		1			1				2	3			
4 3 11	G			7	13	3		2	2		2								1
	M			2	1	1		2											
	+			4	11	1													
	−			4	12	2													
	+			8	7			1											
	−																		
	+																		
	−			5	14	5		2											

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
4 / 11 / 12	G		3	14	10	3		4			6					5			
	M					1		2			3								
	+																		
	−		7		4	4					3					5			
	+		6	15	4			7			6								
	−		2			2		2			5								
	+																		
	−		3	12	10	2		5			6					4			
4 / 12 / 13	G			10	10	1	1	7			7	2							
	M				4			2	3		2								
	+																		
	−			7	14	2	2	6			8								
	+			8	5	1		5			4								
	−			1	4	1	1	2			4								
	+																		
	−			10	7	1		7			6								
5 / 4 / 13	G				3	2	2	3	5	4	1								
	M							5		6									
	+																		
	−							2		4									
	+							5		3									
	−																		
	+																		
	−				6	4		1		3									
5 / 13 / 14	G				10	14	2	5		4	5								
	M							2		1	2								
	+																		
	−				9	11	2	2		3	5								
	+				1	5		5			2								
	−																		
	+				2	9	2	2		4	2								
	−				10	14	2	5			5								
5 / 14 / 15	G				2	14	3	8		7	9								4
	M					3	3	2			7								4
	+																		
	−				4	12	6	3			8								
	+					6		5			4								2
	−																		
	+				2	10	2	3			2								
	−				1	12	2	8			9								4
6 / 5 / 15	G				6	13	3	11	9	21	1								6
	M				1	5	8	8	21	10	5								6
	+				3	5		10	6	16	1								6
	−				6	13	3	11	9	21	1								6
	+							13	6	18									10
	−				2	5	1												
	+																		
	−				9	21	6	4	4	8	1								

Triangles and Baselines	Row	550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
6 / 15 / 16	G				3	16	4	8	3	5	5								7
	M							1		2									1
	+																		
	−				3	12	6	4		2	6								5
	+					9		10			1								6
	−																		
	+					9	1	3											4
	−				3	15	3	6	5	5	4								7
6 / 16 / 7	G				2	14	1	6	1	2	6								4
	M					2		1	2	3	6								3
	+																		
	−					20		6	2	4	9								8
	+					5		4			3								2
	−																		
	+					11		6	1	2	5								4
	−					8		2			1								
7 / 16 / 17	G		1	13	15			3			5				12	7			5
	M							3							2				
	+			9	10			2							10	7			
	−			7	3			3			4				9	5			
	+		2	7	10			3			2				8	6			
	−		1																
	+																		
	−			13	17			2			6				12	5			
7 / 17 / 8	G	5	2					2			3				1	5			
	M														2				
	+																		
	−	4	3					3			5				1	8			
	+	6						2								1			
	−																		
	+	3		2							1				1	4			
	−	1						1											
8 / 17 / 18	G	6				3		1			1			2					
	M	2									1								
	+	3				2					1								
	−	2				2		1											
	+	8				3		1			1								
	−																		
	+																		
	−	8				3		1			1								
8 / 18 / 19	G	3	7			6			10	3				1			6		
	M		3			1			2	11				3					
	+																		
	−		3			5			7	5				1			6		
	+		6			3			8								1		
	−																		
	+		6			3			6	1							1		
	−		6			5			7					1			6		

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END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965

TRIANGLES AND BASELINES		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
9 8 19	G	1	4			16			11					3			13		
	M								1	6				1			2		
	+	1	4			13			14					3			12		6
	−		3			15								3			13		
	+	2	2			4			1					1					
	−																		
	+																		
	−					14			4										
9 19 20	G	2	8		5	5			7	12				2			2		
	M		1		2	5			2	1				4					
	+																		
	−		6		5	5			6	10				2					
	+	2	5		4	3			6	12				1					
	−					1													
	+		6		4	4			6	11				1					
	−	2	5			4			5	1				2					
10 9 20	G		7			10		1	8					3			7		
	M		2					2	3										
	+		7			9			8								7		
	−		3			4		1	2								3		
	+		7			4		2	5								2		
	−																		
	+					1													
	−		4			8			5								5		
10 20 21	G		6			6			7								4		
	M					4			3	1				1			3		
	+																		
	−		2			6			3								8		
	+		3			2			6								2		
	−					1											1		
	+		2			3			2								3		
	−		6			6			5										
11 10 21	G		4	2		5		1			3					4	2		
	M		1					1						3		1			
	+		1			4		1			2					2	1		
	−		2			1										2			
	+		6	3		4		1			2					6			
	−																		
	+																		
	−		2	3		5		1			4					4	3		
11 21 22	G		9			5			3					5			9		
	M		1			3			4								1		
	+																		
	−		8			6			3								7		
	+		4			2			1								6		
	−																		
	+		4			4			2								6		
	−		10			5			3								8		

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
12 / 11 / 22	G	5	2			3		3	2		6			2	3	6			
	M	3	3			2		3			5					6			
	+	5				3		3								6			
	−	4	2			3		2			6					6			
	+	6						2			3					6			
	−																		
	+																		
	−	1	3			4		3			5					3			
12 / 22 / 23	G	7				6			3		4			1			9		
	M	1							3					2					
	+																		
	−	5				8			3		2			1			8		
	+	4				3			3		5			1			8		
	−																		
	+	4				4			1		4						5		
	−	6				3			2		4			1			7		
13 / 12 / 23	G	7	6			2		4			6	2			3	8			
	M	3	6					2						1	10	10			
	+	5	1			2		4			7	2			3	6			
	−	5	3			1		2			1				1	6			
	+	4	2					1								5			
	−																		
	+																		
	−	3	7			2		4			7	3			4	5			
13 / 23 / 24	G	3	1			5					6				1	1	8		
	M	1	1			2					1			3			1		
	+																		
	−	5				6					5						6		
	+					1					4						5		
	−																		
	+	1				3					6						5		
	−	2	2			5					4						7		
14 / 13 / 24	G	3	9			1		4			5			1	7	10			
	M	1	7					2			2			2	4	5			
	+	3	4					3			3			1	4	10			
	−	2	9			1		3			4				7				
	+	4	10					4			1			2	11	9			
	−																		
	+																		
	−		4			1		1			5				1	4			
14 / 24 / 25	G	2	13	6		5		1	2		6			3	4	8	6		
	M			2					1					1	4	3			
	+							1											
	−		6	1		4		3	3		3			3		1	9		
	+	4	11			3		3	1		6			3	1	4	2		
	−		1			1											3		
	+		3			5		2	2		5						6		
	−	1	12	9		4		3	1		6			2	5	12	3		

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TRIANGLES AND BASELINES		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
15 14 25	G	6	22	17		1		6			7			4	14	11			
	M	6	2	1				3						2	3	2			4
	+	3	5	1				5			7			3	10	8			
	−	5	16	13				4			2			2	12	9			
	+	5	14	9				3			2			3	7	8			
	−	2																	
	+																		
	−	3	21	20			2	5			7			3	12	10			
15 25 26	G	6	3					3			3	2			4	6			
	M														1	1			
	+																		
	−	4	3					4			3	3			4	7			
	+	7						3			2				1	1			
	−																		
	+	2						2			3	1							
	−	4	2					1			3	1			4	5			
15 26 16	G	1	15	16				4			6				13	7			
	M	1	2					3			1				3	1			7
	+	1	9	16				1			4				5	2			
	−																		
	+		9	2				3			4				11	7			
	−		13	16				4			5				11	7			
	+	2	13	10				4			3				14	9			
16 26 17	G	7	7					3			4	1		1	4	6			
	M																		
	+																		
	−	6	4					4			4	2		2	4	6			
	+	7	1					3			4			1	2	1			
	−																		
	+	4	6					1			3			1	2	4			
	−	2									4								
17 26 27	G	2			6				1		4						2		
	M	1																	
	+	2			4						2								
	−				5					1	4								
	+	3			3						2								
	−																		
	+																		
	−	1			7				2		4								
17 27 18	G	6			4			6	12	1				2		5			
	M	1			2			2								2			
	+	2			3			2								2			
	−																		
	+	8			3			10								6			
	−																		
	+	4			2			2								4			
	−	3			3			3								1			

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Table: END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965

TRIANGLES AND BASELINES		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
18 27 28	G		6		7									2			1		
	M		2		2												.5		
	+		2																
	−		5		6												1		
	+		8		3												2		
	−																		
	+																		
	−		4		8												1		
19 18 28	G	4	11	3	13	4				1				2		5	4		
	M		1		5	2								4		5			
	+	4	9		13	3				1				2					
	−		6		2	4													
	+	5	5		4	1				1				2		4			
	−																		
	+															2			
	−		10		10	5													
19 28 29	G	5	24	26	9								1	1		3.5			
	M	7	7	9	4								3	6		5			
	+	1																	
	−		15	18	9											2			
	+	6	13	14	2								2						
	−																		
	+	5	21	20	4								1	1					
	−	4	23	24	9								1	1		3			
20 19 29	G	1	14	3	8									6		4	2		
	M	2	1	9	3														
	+	1	16	6	8														
	−		5																
	+	1	6																
	−																		
	+																		
	−		6	1	6														
20 21 30	G	2	6		9	4			8	12				3					
	M	2	1		2				5	2				3					
	+	1	6		13	4			6	12				3					
	−	2	2						2										
	+	2	2		3				4	6									
	−		4		3	4			4	7				1					
	+	4	6		7	1			11	13				5					
	−																		
21 22 30	G	1	5		4	8			8	4							8		
	M	1	5		1	7			6	7							1		
	+	1			4	6											6		
	−		4														7		
	+	1	6		6	11			5	10							12		
	−	1	3		1	2			7	1							3		
	+																		
	−																		

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
22	G		5		3	7			5	9							6		
23	M				2				1	1							3		
	+		4		4	6			4	9							5		
31	−		4			3											4		
	+		5		3	9			5	8							9		
	−		2			3			3								1		
	+																		
	−																		
22	G	5	8		11	4			3	12				3			1		
	M	1	1		4	1			5	1			1	5					
30	+								1										
	−		6		8	4				7				1					
31	+	9	5		6				6	9				1					
	−																		
	+	5	8		11	2			2	12				3					
	−	3	8		7	5			2	11				3					
23	G		3		2	6			5	6							7		
	M									2				1			2		
24	+		3		2	5			5	12							5		
	−		1			4											5		
32	+		3		2	5			2								6		
	−		2			5			3								8		
	+																		
	−																		
23	G	3	6		4	2				11							1		
	M		1		7	3			1	4				3					
31	+																		
	−		5		3	3				8									
32	+	5	5		4	1				11									
	−																		
	+		4		3	1				6									
	−	2	4		3	1				8									
24	G	3	7		6	4			7	12							2		
	M	3	2		3	1			2	2				6					
32	+																		
	−		7		5	5			4	10									
33	+	2	3		3				5	5									
	−																		
	+	3	7		6	2			6	12									
	−	3	6		4	4			5	10									
25	G		4			7	1		7	1	3						5		
	M		3			1	2		3	2				1			1		
24	+		4			7	1		9	2	3						5		
	−		2			3	1		1	1	3						3		
33	+		3			2	1		2		2						1		
	−																		
	+																		
	−		1			7			1								2		

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
25	G	2	7		2	7			9	5							7		
	M					1			2	5			1				4		
33	+																		
	–		2			6			1								2		
34	+	3	6		2	2			8	5							3		
	–																		
	+		2			5			4	3							2		
	–	2	7		2	6			9	4							8		
26	G		4			5		2	1		3								
	M		2			5		2	2		3								
25	+		3			5			1		2								
	–		2			3		1	1										
34	+		6			3		3	1		3								
	–																		
	+																		
	–		2			5		1	1		2								
26	G	3	13		1	9		1	16	4				1	4		12		
	M	2				3			3	7				2	2		2		
34	+																		
	–		8		2	10			12	2					2		9		
27	+	3	8			2	2		11	3				2	3		5		
	–																		
	+	3	10		1	10			9					1	4		12		
	–	3	12		1	6		1	14	4				1	1		10		
27	G	2	10	4	14	1											11		
	M		2	7	4														
34	+	1	6		4														
	–	2	12	8	16														
35	+		5		1														
	–																		
	+																		
	–	1	4	1	8														
27	G	5	14	17	8									7		3			
	M	5	10	13	4														
35	+																		
	–		4	6	5														
28	+	6	6	9	1														
	–																		
	+	5	14	17	8														
	–	5	14	17	7														
28	G	39	32	11								2	25	40	24	1			
	M	1		1									3	3	2				
35	+	6	19	4										12					
	–	5.5	34	12								4	33	46	27	1			
29	+	67	60	15								3	35	74	38	1			
	–	1																	
	+		3	6										4	5				
	–		1	2															

Table header: **END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965**

Triangles and Baselines		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
29	G		3	7										4	1				
	M		11	5															
30	+	1	1	.5															
	−																		
36	+		3	7															
	−		3	7															
	+		2	2															
	−																		
30	G	5	9	9	10									3		2			
	M		5	5	6														
20	+	5	10	13	10														
	−		1																
29	+																		
	−	4	5	5	7														
	+																		
	−	1	6		6														
31	G	8	23	21	5									10	1	6			
	M	2	3	2									3	5					
30	+	8	21	18	4									10					
	−	3	8											5					
36	+	16	36	30	6									19					
	−																		
	+																		
	−		4	2	2														
32	G	3	15	16	16									5		5	1		
	M	1	2	2												1			
31	+		10	13	14									3		2	1		
	−	2	4		3									2					
36	+	5	11	2	8									4		5			
	−																		
	+																		
	−	3	14	10	16									5		5	2		
33	G	1	10	22	7									2		8			
	M	2	1	1										10					
36	+																		
	−			7	3														
35	+	2	14	15	3									4					
	−																		
	+	1	10	19	6									2					
	−	1	10	22	5									2					
34	G	7	10	11	11									4		3			
	M	4	7	6	1								1	2		6			
33	+	5	10	18	12									5		3			
	−	5	3																
35	+	5	2	1	3														
	−																		
	+																		
	−	1	6	3	7									5		4			

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TRIANGLES AND BASELINES		550	580	610	640	670	700	730	760	790	820	850	880	910	940	970	1000	1030	1060
35 36 29	G M + −																		
	+ − + −																		
36 32 33	G M + −	5 2	16 5 16	16 6 18	15 18	1 2								.3 .5 .5		7			
	+ − + −	3 4 2	4 16 4	2 16	4 15 4	1								.3 1					
	G M + −																		
	+ − + −																		
	G M + −																		
	+ − + −																		
	G M + −																		
	+ − + −																		
	G M + −																		
	+ − + −																		

END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965

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X-22

BASELINES		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
1-2	G	5										7	8						
	M			28	1								7	12	21	10	3		
	+																		
	−																		
1-3	G	21	3	11									19	20	7				
	M	10	21	12									7	8	10				
	+																		
	−	7											5						
1-4	G	8											3	15					
	M	10	9	2															
	+																		
	−	3																	
1-5	G	9	3										19		14				
	M		1										10						
	+		2																
	−	5	1																
1-6	G	40	10										6	8		14	6		
	M		1														6		
	+																8		
	−	11	3										2	8					
1-7	G	5										15	3	8			5		
	M	2											6						
	+																		
	−																		
2-3	G						2	4			12	5	1	5	1	1	9	11	6
	M	1	1	1															
	+							1						2					2
	−						2	4			15	4	1	2	1		7	9	6
2-7	G						6	4				2		2			5	6	8
	M			1			1						3						
	+																		7
	−						6	4						1			2	4	4
2-8	G							6						8			2		2
	M						9			4				7					
	+																		
	−																3		1
2-9	G			7				3			15					7			
	M			2	3	1		2	1		5				6	6			5
	+																		
	−			3							1								
2-10	G							3	6	7				2	1	4			5
	M							1	4	3									
	+																		
	−							5		10									2
3-4	G	2		3							11	8	9	2			1	2	
	M										4	4	2						
	+																		
	−										18	8	6						

END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965

5175A-VC-1

BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
3-10	G			11			2	10							8		6		7
	M			6			12	3							8				3
	+																		
	−			3				3											1
3-11	G							5	13	8		2		2	3	2		2	18
	M								5			3							5
	+																		
	−								10	8									3
4-5	G	22	6								2	19	25	13				5	
	M	13	14										1	1					
	+																		
	−	12									2	19	11					5	
4-11	G			9			1				16	2			4		8	6	
	M			9			6				7							1	
	+										12								
	−																		
4-12	G				10						5	17		2	8	4	8	6	11
	M				8						1	2			8	5	1		3
	+				9						8								
	−																		
4-13	G			4									1				1	7	10
	M			1															
	+																		
	−																		11
5-6	G	60	21									13	26	19					
	M													3					
	+																		
	−	82	19									13	26	17					
5-13	G			3							12	17	2	4	13	3	5	7	
	M			4							5						16	3	
	+										19								
	−																		
5-14	G			10	7							5			3	10	12	6	
	M			3														3	
	+																		
	−																		
5-15	G			5									1	2	14				
	M			13															
	+																		
	−																		
6-7	G	1		6									1	5	3				
	M												1						
	+																		
	−																		
6-15	G										17	9	1	4		9	12	11	
	M											4	5	3		6	7	3	
	+										6	11				2	3	5	
	−																		

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BASELINES		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
6-16	G	1		19								14	2	1	14	8	16	14	
	M			7								1			4	6	5	1	
	+															4	1		
	−			5											1				
7-8	G			6	2		9	15	3	3	3			1		8			9
	M			2	3		3	3	4	2									6
	+																		
	−				1			5	6										1
7-16	G						6				2	1	1	2			3	5	5
	M						7				2	1					2	3	5
	+																		
	−																		1
7-17	G			8	1	5	6	8		4		2		1	12	3	6		7
	M			2		6	1								3		2		
	+							2											
	−			7											2				1
8-9	G	2					3		3	7					3		2		1
	M	1							1	1					2				
	+														1				
	−	1							3	12									1
8-17	G						1												
	M																		
	+																		
	−																		
8-18	G	1	1	10	3	1		6			3	5		12	1				
	M	5									3	2		1	1				
	+													1					
	−			5	3			1						4	2				
8-19	G	3	2		1	4		3	10							4	2		
	M					5			6							4			
	+															3			
	−								3										
9-10	G				1	3	2	2				2		4	3				2
	M					3									3				
	+											2		4					
	−					3						1		2					1
9-19	G	5	5	9		10				1	2			7	1		2		
	M	2	1								4			5	7				
	+			1							1								
	−			12										7	2				
9-20	G	6	2					6			2	5			6		1		
	M	5	1					1				5							
	+														1				
	−	1													8				
10-11	G																		
	M																		
	+																		
	−																		

END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965

BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
10-20	G		2	9		7				13	6				8	1	9		
	M+			2		8				4					4				
	−									6					7				
10-21	G	1		4	4	2			9					4					1
	M+	3							1										
	−			4					5					1					
11-12	G						4	7	4								1		3
	M+																		2
	−						2	6	4										3
11-21	G			2	2				2			6		3	10	3	7		7
	M+				1				4					4	4	3			1
	−				4				3						4				
11-22	G			5		9	1		10	1						3			
	M+			1					2							1			
	−			1					4							4			
12-13	G						7	8	5								1	1	
	M+								2								2		
	−						4	4											
12-22	G			1	12		2		5	4	6	7		6	6	8	2		5
	M+				1					2		1		1	3	2			
	−				11					5					4	2			
12-23	G					3			13	8							3		
	M+								2	1							1		
	−								9	11									
13-14	G			1			6	10				2					1	5	5
	M+			1			2					3					1		
	−						3	8				1						5	3
13-23	G			7	5					8	2	4		5	6	4			4
	M+				4					2		7				7			
	−				6					3									1
13-24	G							1	1	14						1	1	6	2
	M+									4									
	−									8									
14-15	G			1			2	6				8			2		2	8	4
	M+											1					2	3	
	−						2	7				7			2		2	6	4

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Table: END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965

BASELINES		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
14-24	G			13	4	6	2				14				13	5			2
	M			1	1	5					4				1	3			1
	+										11					6			
	−			5	8										9				
14-25	G				5		5		2	12	1					8			5
	M			1	2			1								9			
	+																		
	−				5				2	16									3
15-16	G																		7
	M			1									2						
	+																		
	−																		7
15-25	G			11			1	1	3		9				9				5
	M			2		13	3			2	4				3				
	+										11								
	−			9					1						4				1
15-26	G							1	11	6	2	1				4			9
	M				5			1		3	2					3	1		3
	+																		
	−							2	10										3
16-17	G			7	4			2	13	3	6	1	2			7			11
	M					1	1			1	4		1			1			1
	+															3			
	−			2	5				8	6			1						6
16-26	G			9	8	7	2		3		6	1			13		7		5
	M			1	5	7				2	1				1				
	+				1	2					12						3		
	−			7					1						7				
17-18	G		1	9	7				3	16						2	11		3
	M		5		3				1			1				1	1		1
	+				4				3					6		4	4		
	−		1	3						12									
17-26	G						1	2											1
	M																		
	+																		
	−																		
17-27	G		2	9	6				4			4	9	4					
	M	8	2		2				3		3		2						
	+			1															
	−			1	8														
18-19	G	8	11			1						4	10	2					
	M											1	2			1			
	+	8	8			1						2	9						
	−																		
18-27	G		6																
	M																1		
	+												2						
	−		4																

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BASELINES		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
18-28	G					17			1	12		5					3	5	1
	M					4				11							2	3	
	+					4											1		
	−																		
19-20	G		4	2								1	12	6	1		6	2	1
	M		4	2										1			6		
	+												7	9	1				
	−																		
19-28	G							7	3								1	1	4
	M							5										1	1
	+																		
	−																		
19-29	G				2	25						1						3	7
	M					4												4	
	+					11													
	−											2							
20-21	G	2	4	1										7					
	M	2	1	1															
	+	2	1	1										8					
	−																		
20-29	G						3											9	6
	M																		
	+																		
	−																		
20-30	G		2			2	2					5	3				4	5	
	M					1												2	
	+		3			2							3				6	6	
	−																		
21-22	G																		
	M																		
	+																		
	−																		
21-30	G	14	11	3	5	15			9	9	4	18	16	9			15	7	1
	M	3	1		4	5				2		7	5	1			2	1	
	+	7			10							4	5				14		
	−	1								11		5		1					1
22-23	G	2																	
	M																		
	+	2																	
	−																		
22-30	G		9	2		4						5	2	12	2		1		
	M			3										2					
	+			2		2													
	−		8											11			2		
22-31	G	15	10	3	4				9	2	5	6	1	7			17	3	
	M	3			5					4		11	7	1			4	2	
	+	6			8												13		
	−		1						1	3				5					

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BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1,1965																	
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
23-24	G														4				
	M																		
	+																		
	−																		
23-31	G		2	16	3	12								7			3		
	M		1	3									3	6					
	+																		
	−			12															
23-32	G	13	7	3					9	1	1	12	3	3		13	6		
	M	5								1				2		2	2		
	+	5											1			11	5		
	−	1		3					2	2									
24-25	G					4						1							
	M					5													
	+																		
	−											1							
24-32	G	7	5	19		10				8	4	1		15	6		7		1
	M	1	3	1		2				5				2					
	+					4											3		
	−			12						8				5					
24-33	G	8	2	2					2		1	1				8			
	M		3													6			
	+															4			
	−																		
25-26	G			4				2											
	M																		
	+																		
	−																		
25-33	G	4	6	14	7	9			2	4	1			2		3	3		1
	M			3	5					8						4			
	+																		
	−			4	10														
25-34	G	3	1	10		4								4		1			
	M	2		1		4								1					
	+																		
	−			6															
26-27	G	1	1	2		11				9		2		3			3		
	M	4		1		7				3							1		
	+					1													
	−									8				1					
26-34	G	5	3	2	7				5		3	6		9	9	4			
	M			2								1		1	3	1			
	+													3	9	5			
	−				3				1										
27-28	G					6	6		6			1	4	8			12	5	
	M					9	1										3	5	
	+					1											1	1	
	−																		

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BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
27-34	G/M	3	4			1	1					10	10	6			9	6	3
	+	1										3							2
	−	2	4									8	10	6			9		3
27-35	G/M									1								6	4
	+																		
	−																		
28-29	G/M				1	30	35	31	5										3
	+							1	1										
	−				2	37	35 (4)	9											1
28-35	G/M					9	41	29	15								1	29	27
	+						1		6									7	7
	−					9	61	30	11								1	31	27
29-30	G/M					1	7		12									20	11
	+																	2	15
	−																	4	
29-35	G/M						1		1										
	+																		
	−																		
29-36	G/M						9	13											1
	+						4												2
	−							3											
30-31	G/M	2	1				3					3	3				2	10	8
	+						3		6										3
	−	2	1				6					3	3				2	10	7
30-36	G/M				11	13	19												1
	+					4													
	−					16													
31-32	G/M		2	3									3	4			7		
	+													1					
	−		2	3									2	4			7		
31-36	G/M				7	17	27	2	7	1	3	6					7	19	26
	+					1	4	7	21			5							3
	−				14	5	18	2	11			9						2	2
32-33	G/M	2	7						2			5	15	2			5	2	3
	+		6									5	13	1			3		
	−	2	1									2	8				2		

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BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1,1965																	
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
32-36	G								9	4								10	22
	M								2	8									
	+																		
	−																		3
33-34	G	8	4									12	18	12			7		
	M																		
	+	5	4									6	14	14			7		
	−	3										2							
34-35	G					2												4	1
	M					1													
	+																		
	−																		
35-33	G				10	16	12					1					5	6	15
	M				2		4					4						1	1
	+				20	8													
	−											1							1
35-36	G			4	2	1	37	34											
	M																		
	+						8												
	−					1	37	24											
36-33	G																	1	7
	M				2														
	+																		
	−																		
	G																		
	M																		
	+																		
	−																		
	G																		
	M																		
	+																		
	−																		
	G																		
	M																		
	+																		
	−																		
	G																		
	M																		
	+																		
	−																		
	G																		
	M																		
	+																		
	−																		
	G																		
	M																		
	+																		
	−																		

TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																		
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
1 2 3	G M + −	3										5	5 3 / 1	3					
	+ − + −											5 / 6	4 / 5						
1 3 4	G M + −	2	1																
	+ − + −																		
1 4 5	G M + −	5	2																
	+ − + −																		
1 5 6	G M + −	11 / 9																	
	+ − + −	3 / 11																	
1 6 7	G M + −												14						
	+ − + −																		
1 7 2	G M + −												4						
	+ − + −																		

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
2 7 8	G M + −							4	4		1	10		1					1
	+ − + −																		
2 8 9	G M + −			7 3 4					7 3 3										1 1
	+ − + −			6 7					7										2
2 9 10	G M + −							1 4	8 4 7			6			1				2 1 3
	+ − + −							2	10										1
3 2 10	G M + −							7				2 9 2 2		2					1 5 1 1
	+ − + −																		
3 10 11	G M + −																		2
	+ − + −																		
4 3 11	G M + −											11					6		
	+ − + −																		

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
4 11 12	G																		
	M																		
	+																		
	−																		
	+																		
	−																		
	+																		
	−																		
4 12 13	G			2								1					2	3	6
	M			6								2						3	3
	+																		
	−			3														4	7
	+			2														1	3
	−																		
	+																		1
	−			2								2						3	6
5 4 13	G	1										2	1					3	
	M											10	2						
	+											2	1						
	−																		
	+											3						1	
	−																		
	+											2	1						
	−																		
5 13 14	G											4							
	M			6								5					1		
	+																		
	−																		
	+											2							
	−																		
	+											6							
	−																		
5 14 15	G																1		
	M																		
	+																		
	−																		
	+																		
	−																		
	+																		
	−																		
G 5 15	G	2										5							
	M	3										3	11						
	+	2										3							
	−	2										5							
	+																		
	−																		
	+																		
	−	1										6							

51750 VO 2

TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																		
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
6 15 16	G											14		4	3		3	5	
	M																1	1	
	+																		
	−											9					3	5	
	+											12					3	4	
	−																		
	+											7							
	−											13					3	5	
6 16 7	G												1						
	M																		
	+																		
	−																		
	+																		
	−																		
	+																		
	−																		
7 16 17	G							4	2			5							2
	M							6	1			6		1					
	+							4	1			3							
	−							4	1			5							
	+							3				4							
	−																		
	+																		
	−							4	4			6							
7 17 8	G								2										
	M								2										
	+																		
	−								3										
	+								2										
	−																		
	+								1										
	−																		
8 17 18	G			5					4										
	M																		
	+																		
	−			3															
	+			3															
	−																		
	+																		
	−			7															
8 18 19	G	4	5	5					2										
	M		5						4										
	+		3	7					2										
	−		4																
	+		5																
	−		5	5					2										

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
9 8 19	G	1	2	16		1								9			8		
	M			3		3											2		
	+			15		1											8		
	−			8													3		
	+			9		1											4		
	−																		
	+																		
	−			8													1		
9 19 20	G	4	10	2		1											2		
	M	1	7														2		
	+																		
	−	3	11																
	+	4	6														2		
	−																		
	+	3	10														2		
	−	4	9														1		
10 9 20	G		1	5					4	1						6			
	M		2	6					4	1									
	+																		
	−								4										
	+																		
	−																		
	+																		
	−			3					4	2									
10 20 21	G			6													2		
	M			3		3											1		
	+												6						
	−			11															
	+			6													1		
	−																		
	+																		
	−			3													2		
11 10 21	G																		
	M																		
	+																		
	−																		
	+																		
	−																		
	+																		
	−																		
11 21 22	G			9													1		
	M			5															
	+																		
	−			9															
	+			5															
	−																		
	+			9															
	−			8															

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TRIANGLES AND BASELINES		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
12 / 11 / 22	G			5					4					1					3
	M																		
	+																		
	−																		
	+																		
	−																		
	+																		
	−																		
12 / 22 / 23	G			6													1		1
	M			5													2		1
	+																		
	−			7													1		1
	+			7													2		1
	−			6															
	+																		
	−			2													1		1
13 / 12 / 23	G			7															3
	M			5															
	+			7															
	−																		
	+																		
	−																		
	+																		
	−			7															
13 / 23 / 24	G																		2
	M																		
	+																		
	−																		
	+																		
	−																		
	+																		
	−																		
14 / 13 / 24	G			3					7								1		4
	M			4					8										
	+			3					6										
	−			2					7										
	+			4					8										
	−																		
	+																		
	−								1										
14 / 24 / 25	G						1	1	1		1	4		5					5
	M						3		1			2		3					1
	+																		
	−								8			3		6					2
	+						1		8		1	4		5					5
	−																		
	+								7			2		5					
	−						1		9		1	4		2					4

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TRIANGLES AND BASELINES		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
15 14 25	G			2			2	8	10			1			5	1	4		4
	M						5	4	1			6			3		4		2
	+			2			2	5	9						5		4		3
	-							6	8			1							2
	+							5	2			2							
	-																		
	+																		
	-			4			3	10	12						6		5		8
15 25 26	G								4										2
	M								1										
	+																		
	-								5										
	+								3										
	-								1										
	+																		
	-								4										
15 26 16	G							5	3			5		2	3	2			4
	M							12	5			6							
	+																		
	-								4										
	+							5	1			5							
	-							5	3			5							
	+							8	1			8							
	-																		
16 26 17	G			2					6										
	M			1															
	+																		
	-			4					8										
	+			1					7										
	-																		
	+								3										
	-			1															
17 26 27	G			7					2					2					
	M																		
	+																		
	-			6															
	+			3															
	-																		
	+																		
	-			10															
17 27 18	G		4	3					3										
	M	1	1	2															
	+																		
	-		1	4															
	+		7	3															
	-		4																
	+		3																
	-			3															

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TRIANGLES AND BASELINES		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																			
18 27 28	G																2		
	M																2		
	+																2		
	−																		
	+																3		
	−																		
	+																1		
	−																		
19 18 28	G					1	3										6		
	M						1										4		
	+					1	3										6		
	−																		
	+					1	3										3		
	−																		
	+																		
	−																		
19 28 29	G						1										4	12	15
	M						14										3	9	
	+																	10	
	−																		
	+																2	2	
	−						1										4	12	
20 19 29	G					2	6											3	
	M																		
	+																		
	−																		
	+																		
	−																		
20 21 30	G	3	9			2						5					8		
	M	4	9			6						1					3		
	+	3				2						5					8		
	−																		
	+	1	2														2		
	−	2	6									1					4		
	+	4	6			4						8					12		
	−																		
21 22 30	G	5	8	15										6			5		
	M	4	7	13									2	5			5		
	+																		
	−			8													3		
	+	7	10	10										8			7		
	−																		
	+	2	3	8										2			1		
	−																		

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
22 23 31	G M + −		3 2 3	10 1 9 2													3 3 3		1
	+ − + −		4	12 3														4		
22 30 31	G M + −	3	4 6			2 5						4	5 7 4	3			11 3 7		
	+ − + −		4 2			3 2 2							3 5 5				10 11 9		
23 24 32	G M + −	1	1 3 1	4 4										2		1	1		
	+ − + −		2	3 3																
23 31 32	G M + −		8 6 6			3							3	1			3		
	+ − + −		8 6 6																
24 32 33	G M + −	2 5 4	10 8 11			3											3 7 3		
	+ − + −	2 2	1 10 9														3 3		
25 24 33	G M + −	2	4 2 4	1 1 1					5										
	+ − + −			1															

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
25 33 34	G M + −	4	9	4 3		5								9			4 6		
	+ − + −			4 4													3 1 4		
26 25 34	G M + −	1							1 4 1 1					1					
	+ − + −								2 1										
26 34 27	G M + −	8	3 13	13 13					3 3	5 2 6				7		1 3	5 1 1		1 2
	+ − + −		5 2 3	2 12 13					2 3 3	5 4						2 1 1	4 5 5		1 1
27 34 35	G M + −																		1
	+ − + −																		
27 35 28	G M + −																	6	
	+ − + −																		
28 35 29	G M + −					9 9	10 11												
	+ − + −					18	14 4												

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
29 30 36	G M + −																		
	+ − + −																		
30 20 29	G M + −																		
	+ − + −																		
31 30 36	G M + −					15 15 6	4 8 3 2										3 1	6 4 5	1
	+ − + −					30	6										2	9	
32 31 36	G M + −					4 1	5 5	2										7 3 5	2 2
	+ − + −					5 4	3 5											4 8	1 3
33 36 35	G M + −																	9 7	11
	+ − + −																	10 9 5	
34 33 35	G M + −					6 8 6	3 3										2	8 3 12	
	+ − + −					4	3											6	

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		1090	1120	1150	1180	1210	1240	1270	1300	1330	1360	1390	1420	1450	1480	1510	1540	1570	1600
35 36 29	G M + −																		
	+ − + −																		
36 32 33	G M + −								1									6 10 12	3
	+ − + −																	8	
	G M + −																		
	+ − + −																		
	G M + −																		
	+ − + −																		
	G M + −																		
	+ − + −																		

BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
1-2	G						17	10	15	15	10								7
	M						5	13	9		1								
	+																		
	−																		
1-3	G					2	15	6	15	27	9								
	M					3	10		2										
	+						2												
	−																		
1-4	G						6	1											
	M																		
	+																		
	−																		
1-5	G							1	17	10									
	M							24	12										
	+									1									
	−																		
1-6	G						4	4	29	37	2								4
	M							25	20	4	1								12
	+							6	35	38									7
	−								3	7									
1-7	G							6	13	7									8
	M									6									
	+																		
	−																		
2-3	G			9	18	11				8	1	6					1		3
	M				7	1						1							
	+				17	7						7							1
	−			11															
2-7	G				8	11			4		1	3							
	M										1	3							
	+				7	8					1	1							
	−																		
2-8	G		3	7	4		6		3					9	4	3	6		3
	M			2	2		3		3					10					3
	+			6					1					12					1
	−																		
2-9	G			3			3	1	4						3	2	2	2	10
	M						4	7								9	2	9	2
	+																		3
	−																		
2-10	G	1					2	2				3			1	4	1		2
	M	4					3	1					2			2	6		
	+							1							1				
	−	2																	
3-4	G				2	1			4	3	10								
	M									1									
	+										1								
	−				2					1	8								

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Table header: **END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965**

BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
3-10	G		1	2	6		11	2	10			5		3	9	9	3		6
	M			2	3		6		5					1	1	3	4		5
	+			4					1					6		4			3
	-																		
3-11	G	4			5	2	4	2			10	3				13	2		4
	M	4			1	7	4	1			4	3				2	6		
	+																		
	-	6									5	5				2			2
4-5	G								5	25	8								
	M										1								
	+																		
	-								4	19	8								
4-11	G			5	12	7	2		3		2								3
	M			6	3		4		5		2								
	+			2	2														
	-																		
4-12	G			5	10	6	12	3	9		2	13				1	7	1	11
	M					13	3	4			1						6		2
	+							4											5
	-											6					2		
4-13	G				7	7	3		1			7	3			5	3		
	M				1	2	3		1		1	5					2		
	+																1		
	-				3	4						9				5			
5-6	G								5	30	13	4							
	M				6														
	+								3	5	1	1							
	-								2	26	13	4							
5-13	G				10	1	4		9		1								7
	M					4			2		2								5
	+																		
	-																		
5-14	G			2	13	6	8	6			3	2							10
	M					6	12												5
	+																		
	-																		
5-15	G				9	6			2	8	6	4						1	1
	M				2	12					3	2							
	+				2														
	-										3								
6-7	G					7	4		5		12	10							
	M					4				1		5							
	+										1	2							
	-					1					7	4							
6-15	G				2	6	7	7	13		4								10
	M						1	5	4		2								6
	+							6	5		2								7
	-						5				1								

BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
		\— END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
6-16	G				9	14	21	12	16		13	5							2
	M						4	7	6		1	1							1
	+							6	4										
	−					7	4	2	2		1								
7-8	G	7		2			3	4	3			3			5	7	11		6
	M	7					7	1	2							8			2
	+							2											
	−	4																	
7-16	G				8	1				1	1			1		1	1		
	M										1								
	+													2					
	−				6						1					1	1		
7-17	G		4	9	4		9	1	10		1	7		13	18	8	6		5
	M				1		4		3					7	1	2	4		4
	+			10					2					17					2
	−			5			2					2			4		1		
8-9	G	2			1		1							2	3	1			
	M						1												
	+																		
	−	2												3					
8-17	G								1					1	1				
	M																		
	+																		
	−																		
8-18	G		4	1			11		2					9				1	
	M													1					
	+	1	3				2		2					3					
	−						5											2	
8-19	G	8		3		1		2	6			1	1	5			1	3	12
	M							2						3					3
	+	4				1		4	2				4	1					14
	−																		
9-10	G		1				5								3				
	M														1				
	+						5								3				
	−		1				2												
9-19	G	4	9				6	5				1	2	5				5	1
	M	1				1	5	3			1		2					5	
	+		3				5	8										5	
	−	1																	
9-20	G	5	1					1	4			1		1					4
	M	1	1								1	1							
	+							2	5										5
	−	2																	
10-11	G			1											3				
	M																		
	+																		
	−														1				

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BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
10-20	G	4	3	10			11	4	5			6	4	9			9	2	7
	M	1		1			2		2			2	3	3			1	5	2
	+			3									1	4					
	−						4										2	3	4
10-21	G	5		6			3		5					1					
	M	1		1					1					1					
	+																		
	−	2					1												
11-12	G				2	1								1	5	7	1		
	M																1		
	+																		
	−														3	5	1		
11-21	G		3	10			10		7			7		14	5		8		9
	M			4			4							3	1		5		
	+													9					
	−						2		1										4
11-22	G	7		2			2		7			2					3	2	4
	M	2							1			1			4			3	
	+																		
	−	3							4										3
12-13	G			2	4									1	5	5			
	M				1														
	+																		
	−			2	2									1	4	1			
12-22	G		4	7			8		4			3		18	1		7	2	8
	M		2				2							1			2		1
	+		6											11					4
	−						2												1
12-23	G	9		1			2		6			5	1	2	3		8	1	2
	M	1												1					
	+			1					3					1			1	1	2
	−	5										1	2				1	1	
13-14	G			1	9					1					4	3			
	M				1														
	+																		
	−				8										4	2			
13-23	G		8	2			4	2	1			1	3	10	1		7	1	7
	M		3	1			4	4	1				3	3	2				4
	+		5										5	3					1
	−																		
13-24	G	5	1						4			4	3				9		4
	M	5																	
	+								1										
	−																		1
14-15	G		1	1	10	2	3				2	1							
	M				1	2													
	+																		
	−		1	1	8	1	1					1							

BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
14-24	G	2	11	13			4	6	2				6	12	7	1	3	4	8
	M		2	2				1					2	1				1	1
	+																		
	−		10	9				7					9	9	2			6	7
14-25	G	12					4	4	1				3		1	4	6		4
	M	1						5	1							3	1		
	+																		
	−	17					3						5				5		4
15-16	G	1			1				3	1	3								
	M																		
	+																		
	−	1																	
15-25	G		1	17	1		9	2	7			2	10	7	1	6	3	3	1
	M			3			1						4	1	4	1			4
	+			8									8						
	−			3			3							1		2			2
15-26	G	6					6	4			1	1				2	5		5
	M	3					2	1								4	4		1
	+																		
	−	5						2											4
16-17	G	8		1			7	4	2					4	8	7			6
	M			1			2	5	1					1	2	1			
	+																		
	−	9					2								1	1			5
16-26	G	7	1	18	1		11	12				6	20	14	3	12			4
	M			1	2		1						1	1	3				2
	+			11				7					20	2				1	
	−	6		4			6	1				3		2		8			5
17-18	G	10		12		1	6	22	1			9	2	8	6		9		7
	M	3		1		1	1	1	1				1	2					
	+			2				14				1							
	−	8		5			2	2				3	3	6			4		9
17-26	G			1										1	2	2			
	M													1					
	+																		
	−			1															
17-27	G		4	4			7							9			2	4	
	M		1	1										1					
	+		1											2					
	−						5												
18-19	G	3							2	14	2	2							
	M										1								
	+										1								
	−	3							2	8		2							
18-27	G					3				7	3			2					
	M					1													
	+																		
	−					1				4	1								

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BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
		END OF TIME INTERVAL IN DAYS FROM JAN 1,1965																	
18-28	G	6	11	2		8					13	5	6	10			12	8	
	M	8	9	5		2							9	3			2	2	
	+												3						
	−																	7	
19-20	G					2	2			7	6								
	M					2	1			6	5								
	+						3												
	−																		
19-28	G	3		6		17	10				6	7		5			10	8	
	M	3		7	2	7	1				3	3		1				6	
	+					7	1							2					
	−						12				2	1					10	16	
19-29	G	16	2	3	1	4	4							7			1		
	M		11	15	5		4						1	9			1		
	+																		
	−					1													
20-21	G	1							1		3						2		
	M		2								1								
	+																		
	−	1									1						2		
20-29	G	7	11	12	5	8											15	1	
	M					3						4		12	1		3	4	
	+																1		
	−																	2	
20-30	G			2	2	4			2	2	2				1		2		
	M	1		2		1						1							
	+			1		4			3	3	2						1		
	−																		
21-22	G								1	3	3								
	M									3					3				
	+								1	3									
	−																		
21-30	G	14	3	10		8			12	11	13	10	2	12			11	2	13
	M	2	4			1					1		3	1			1		
	+		5	1					10			2	4	7			3		10
	−	8		1							1								
22-23	G								1		2								
	M									2					2				1
	+								1										
	−																		
22-30	G	1	4			1	9	3		1	4	2		1			2	7	
	M		4				3				1							3	
	+																		
	−						7											8	
22-31	G	9	4	7		3			7	18	5	5	3	7			3	1	8
	M	2	3						4	9	4			1					
	+		3						8	1				4					4
	−	3				1											1		

BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																
		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
23-24	G								1						1				
	M																		
	+																		
	−																		
23-31	G	7	3	2			9		4			5		6			6	2	
	M		4			1	3				9			5			1	4	
	+						1												
	−																		
23-32	G	8	2	5			6	1	11	10	1	5		1			4		11
	M		1						1	4	3								
	+							2	8	6	1								9
	−	2																	
24-25	G		1	1					2							1			
	M																		
	+																		
	−		1	2															
24-32	G	7	11	8			1	11	3	1	5	1	8	2	8		9	5	4
	M		1				2	4	3			3		4	4			6	5
	+													4					
	−		7	4				4					1					8	
24-33	G	5	2					2	6	1		3							6
	M								4										3
	+								3										6
	−																		
25-26	G									1						2			
	M																		
	+																		
	−																		
25-33	G	2	1	2			1	1	1			7	3	5			7	6	5
	M			6			1							3			1		2
	+																		3
	−																		
25-34	G	6					1		9	3		3		1			1	2	5
	M	2								1								7	
	+								5									1	7
	−	1																	
26-27	G	9		1			1	3	12	2		4	2	4			5	1	8
	M	5							1	1		1	2	1					
	+								7			1	2						10
	−	4						1									3		
26-34	G	2	3	4			11						1	11	1		1	4	5
	M	1					2											3	1
	+																		
	−	1	5	2			6							6				6	5
27-28	G	4	4	11		3								4					6
	M					3													
	+																		9
	−																		

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BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1,1965																	
		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
27-34	G					2				14	14								1
	M																		
	+																		
	−					2				13	11								1
27-35	G					5											7	5	
	M	3	6	6	2						1	7		8			2	7	
	+																		
	−																	4	
28-29	G		5	19	14	1							2	12	37	44	25		
	M			1	1									1	1				
	+		6	19	17								3	13	39	49	29		
	−			2										1	8		2		
28-35	G	1		3	7									7	12	16	37	2	
	M																		
	+			3	7									5		2	2		
	−														6	38		2	
29-30	G				6	2						4		7	2	11	21	13	
	M	11		5	3						1	7		4		1			
	+																7	5	
	−															2		14	
29-35	G														5				
	M	3	2	3										15	4				
	+																		
	−														9				
29-36	G														4	4			
	M	2	3											4					
	+																		
	−														4	1			
30-31	G		2	5	3					2				1				1	
	M																		
	+		1	5	3					2									
	−									1									
30-36	G				1										4	1	1	5	
	M																		
	+																		
	−																		
31-32	G					6				1	4	1							
	M																		
	+					5				1	3	1							
	−					1													
31-36	G	18	13	26	29	1							4	20	28	24	8		
	M		9	1										4	6	5			
	+		12	19	9	2								16	16	12	2		
	−	12	3	5	7	2								2			2		
32-33	G			4		10				5	9			1					
	M			1		1													
	+			4		9				3	7								
	−					2				3	1								

BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
		END OF TIME INTERVAL IN DAYS FROM JAN 1,1965																	
32-36	G	12	8	21	19	8					2	7	2	18	2	3	15	7	
	M	9	5	3	6						2	3	4	2	3	4		1	
	+		4	1										4					
	–	1			4						4	1	1					4	
33-34	G	4	3			11	8			17	20	4					1	8	1
	M										1								1
	+	3				9	9			15	15	4					1	10	
	–	2	3							3	2	1							
34-35	G		5	8	3	1							2	7			1		
	M	4				2													
	+																		
	–																		
35-33	G	13	6	20	28									11	6	8	2		
	M	1	7	8	2								1	7	3				
	+		2	6	6									5					
	–	3		5	1														
35-36	G			1	6									3	30	30	14	1	
	M	2											1	4	2	0			
	+				1										10	5	1		
	–				6										27	29	12		
36-33	G	6			4					1	8						11	13	
	M																2	11	
	+																		
	–																10	15	
	G																		
	M																		
	+																		
	–																		
	G																		
	M																		
	+																		
	–																		
	G																		
	M																		
	+																		
	–																		
	G																		
	M																		
	+																		
	–																		
	G																		
	M																		
	+																		
	–																		

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TRIANGLES AND BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
								END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965											
1 2 3	G					4			2	5	12								
	M					4			7		4								
	+																		
	−					1				1	8								
	+					3			2	2	5								
	−																		
	+																		
	−					4			2	7	12								
1 3 4	G																		
	M																		
	+																		
	−																		
	+																		
	−																		
	+																		
	−																		
1 4 5	G																		
	M																		
	+																		
	−																		
	+																		
	−																		
	+																		
	−																		
1 5 6	G								5	5									
	M									1									
	+																		
	−																		
	+									10									
	−																		
	+									5									
	−																		
1 6 7	G								3	7	5								
	M										10								
	+										5								
	−										5								
	+																		
	−																		
	+																		
	−										2								
1 7 2	G								4		4								
	M										1								
	+																		
	−																		
	+										6								
	−																		
	+										4								
	−																		

Triangles and Baselines		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
2 7 8	G			3								2			2	7			
	M			4								3			1				
	+			3								2			2				
	−			3								2			2				
	+			3											2				
	−																		
	+											1							
	−																		
2 8 9	G			9			3		2			5		2	10	2	3		1
	M			3					4			3			9				
	+			2								2			2				
	−																		
	+			4											2				
	−			7					2			4							
	+			9					2			5			13				
	−																		
2 9 10	G			2					1			1			4	2	1		1
	M			3											2				1
	+														2				
	−																		
	+														2				1
	−																		
	+														4				
	−			2															1
3 2 10	G			5	1	1	1					4			4	1			1
	M			3	1							2			3	2			
	+			5	1							4			4	1			
	−			5								4			4				
	+			5								2			4				
	−																		
	+											2							
	−			1	1												1		
3 10 11	G							1				2				2			
	M																		
	+																		
	+																		
	−																		
	+																		
	−																		
4 3 11	G				6	2					1								
	M										4								
	+										1								
	−										1								
	+										1								
	+																		
	−										1								

END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965

TRIANGLES AND BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																			
4 / 11	G			7	3		4		4		2	2							
	M				2							1							
	+																		
	−			2								1							
12	+			8	5						4	2							
	−																		
	+			4															
	−			7	3						2	2							
4 / 12	G				7	1	1		3		4					3			
	M				3		3		3		3					2			
	+																		
	−				2		2		4		6					4			
13	+				2		1		2							1			
	−																		
	+																		
	−				8		1		3		4					3			
5 / 4	G								2		5								
	M										5								
	+										5								
	−										5								
13	+										4								
	−																		
	+										5								
	−																		
5 / 13	G				4	1	6		3		6								
	M				3	1													
	+																		
	−				4	1					5								
14	+				1						3								
	−																		
	+																		
	−				4	1					6								
5 / 14	G				1			1			5	9							1
	M				7			2			4	3							3
	+																		
	−										2	6							
15	+							1				4							1
	−																		
	+										8	4							
	−							1				9							1
6 / 5	G					5			4		11	5							
	M										3	6							
	+										8	1							
	−										11	5							
15	+										7								
	−																		
	+																		
	−										5	9							

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Triangles and Baselines	Type	1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
6 / 15 / 16	G M + −				8 / 6	10 / 1 / 11					6 / 5	8 / 2 / 6							1 / 2
	+ − + −				6 / 8	8 / 9					1 / 6	4 / 4 / 8							2 / 1
6 / 16 / 7	G M + −					3 / 7 / 6			3		4 / 2 / 6	7 / 11							
	+ − + −					3					1 / 4	2 / 6 / 4							
7 / 16 / 17	G M + −			7 / 3 / 5 / 5	1							4 / 4 / 3 / 4			4 / 1 / 2 / 4	4 / 3 / 2 / 3			
	+ − + −			6 / 6								3 / 5			2 / 5	3 / 4			
7 / 17 / 8	G M + −						1												
	+ − + −																		
8 / 17 / 18	G M + −						1		1										
	+ − + −								1 / 1										
8 / 18 / 19	G M + −			1 / 3		1			2 / 3	3		1						3 / 3 / 3	
	+ − + −			1 / 1					3 / 2 / 1									1 / 3	

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TRIANGLES AND BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
9 8 19	G	3		6			4		5	1		3		3			6		1
	M	3		1			3			1		5		1			3		
	+	3		6			5		5	1		3		3			6		1
	−	3		6								3		3			4		1
	+	1		4			1		3			1		4			2		1
	−																		
	+																		
	−																1		
9 19 20	G								5	1				3					
	M	2	2						4			3		1			2		
	+																		
	−								4	1				3					
	+								4	1				2					
	−																		
	+								5	1				1					
	−								5	1				3					
10 9 20	G			2					5			3		2	1		3		2
	M								3					2	2		2		3
	+								5			2		2			3		1
	−											2		2					
	+								3			2		2					4
	−																		
	+											1		1					
	−								3			2		1	1		2		
10 20 21	G	1		1			3		1	1				4					
	M	3					3			2				1					
	+																		
	−						2							3					
	+	1					1		1	1				2					
	−																		
	+						1												
	−	1					3		1	1				4					
11 10 21	G			2					2										
	M																		
	+																		
	−																		
	+																		
	−																		
	+																		
	−																		
11 21 22	G	5		2			2		2			3		3			1		
	M			3										1					
	+																		
	−			1										1					
	+			1										2					
	−			2															
	+			2										3					
	−																		

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1,1965																		
		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
12 11 22	G			3			5		2						3		3		
	M			3					4										
	+			3					2										
	−			3					2										
	+			3					2										
	−																		
	+																		
	−			1					1										
12 22 23	G		1	3					2			1		1	2		2		
	M			2					2			3			1				
	+																		
	−			2					1			1							
	+			4					3			1			3				
	−																		
	+																		
	−			3					2			1			2				
13 12 23	G			6			4		2						5		4		
	M			4					4						3				
	+			5					2						5				
	−			3											2				
	+			2											1				
	−																		
	+																		
	−			6					2						6				
13 23 24	G											2		1	1		1		
	M													1	2				
	+														1				
	−																		
	+													1					
	−																		
	+													1	1				
	−																		
14 13 24	G						2		3			2			1		7		
	M								3			2			2				
	+								3			2			1				
	−								2			2			1				
	+								5			2			2				
	−																		
	+																		
	−																		
14 24 25	G		1	2			5		1					5	2		5		1
	M			1			2		1			1			3		2		
	+																		
	−						6		4					5			4		
	+						4		1					3			6		
	−																		
	+						4		5					4	4		3		
	−						3		4					4			4		

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TRIANGLES AND BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
15 14 25	G			4	4		2		3			8		3	19	7	4		4
	M			3	1		1		3			3		2	1	5			2
	+			2			2		2			6			10	5	4		3
	−			4	2				1			6		3	12	5	1		4
	+			5					4			4		6	7	4			4
	−													1					
	+																		
	−			2	7		3		1			8			14	8	5		2
15 25 26	G			1			3		1						1		2		
	M								1						2				
	+																		
	−								2						2				
	+								1						1				
	−																		
	+								1										
	−								1										
15 26 16	G	1		7	2		1		1			4			6	4	1		4
	M			2								4			4	10			1
	+																		
	−			2								2			2				
	+			6								3			5	4			4
	−			7								4			6	4			
	+			9								4			7	7			6
	−																		
16 26 17	G			1			1		2						2		1		
	M			1			1		1						1		1		
	+																		
	−			2			2		2						3		2		
	+			1			1		3						1		1		
	−			1															
	+								1						2				
	−								1										
17 26 27	G						2		1										
	M																		
	+																		
	−																		
	+																		
	−																		
17 27 18	G			1		1						2					5		
	M																		
	+																		
	−																		
	+																		
	−																		

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
18 / 27 / 28	G M + –																2/3/ /2		
	+ – + –																3/ / /2		
19 / 18 / 28	G M + –		1	1		6/3/6					2/4/2	2		1/1/1	2	1	6/4/6/1		
	+ – + –					4/ / /4					2			1			4/ / /3		
19 / 28 / 29	G M + –	2		3	5/16/ /2											2/2	10/5/ /12		
	+ – + –				5/ /5											2/ /2	1/10/10		
20 / 19 / 29	G M + –					1											4/3/4		
	+ – + –																4		
20 / 21 / 30	G M + –	2/1/2				4/ /4		1	12/3/11/1	5/2/5				4/2/4			3/ /4		
	+ – + –	4				6			9/3/19	1/10				2/6			1/3		
21 / 22 / 30	G M + –	1/4/1/1		3/1/3/3			6/5/2		2/1/2	4/4	5	1/2/1/1		2/1/1			3/3		
	+ – + –	2		5/1			8/3	1	5			2		2/3			5		

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
22 23 31	G M + −	1		5	1	2 / 2				1 / 1		4					2 / 2		
	+ − + −					3			2								4		
22 30 31	G M + −	1 / 5	1		6 / 5 / 4					5 / 4 / 2	6 / 4 / 3	2		3 / 5 / 1			2 / 4 / 3		
	+ − + −	1 / 1 / 1			3 / 6 / 6					3 / 5 / 5	4 / 5 / 6			2 / 3 / 3			2 / 2		
23 24 32	G M + −				1 / 1				3 / 3	1		3					3 / 1 / 3 / 1		
	+ − + −				1 / 1				3 / 3								2 / 4		
23 31 32	G M + −				4					5	3	1		3					
	+ − + −																		
24 32 33	G M + −	5							3	5 / 9 / 5	3	1		1 / 6 / 2			3		
	+ − + −									5 / 5				1 / 1					
25 24 33	G M + −								3 / 4 / 3			1					4		2
	+ − + −								1										

TRIANGLES AND BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
25 / 33 / 34	G M + −	3		4		1	2 5		4 2			1		2 5			3		
	+ − + −						1 2		2 2 4					2 2					
26 / 25 / 34	G M + −						1 2 1		3 1 2 1										
	+ − + −						2 1		4 2										
26 / 34 / 27	G M + −	3		6 4 4		2 2	2 2 3		7 2 6	1		4 3 3		6 2 1			7 5 6		1 2
	+ − + −			2 6 6		2 2	2 2		6 8 9			1 4 4		4 7 3			3 7 7		2 1
27 / 34 / 35	G M + −			1	1	8								3			7 4 6 7		
	+ − + −																3		
27 / 35 / 28	G M + −				6	2								2		1	5 10		
	+ − + −																2 5 2		
28 / 35 / 29	G M + −														8 5 8	23 7 26			
	+ − + −														12 4	28 1 9 1			

TRIANGLES AND BASELINES		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																			
29 30 36	G M + −														6	15			
	+ − + −																		
30 20 29	G M + −				3												3 6 3		
	+ − + −																3		
31 30 36	G M + −			8	11	5									7	7 6 10			
	+ − + −															13			
32 31 36	G M + −			3 1	1	6 4								4 2 1		2 4	11 8		
	+ − + −			2 3	1 1	2 6								4 4		7 2			
33 36 35	G M + −				2									1		9 10	2 10 1		
	+ − + −															11 9 9	1 2		
34 33 35	G M + −	3		1		3 6							7				9 11		
	+ − + −					3											9		

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																		
		1630	1660	1690	1720	1750	1780	1810	1840	1870	1900	1930	1960	1990	2020	2050	2080	2110	2140
35 36 29	G M + −																		
	+ − + −																		
36 32 33	G M + −			2	3 4 6	4 1 2				1				6			1	9 3 11	
	+ − + −				3	1 4												1 9 3	
	G M + −																		
	+ − + −																		
	G M + −																		
	+ − + −																		
	G M + −																		
	+ − + −																		
	G M + −																		
	+ − + −																		

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BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965						
		2170	2200	2230	2260	2290	2320	2340
1-2	G	11		9	18			
	M				1			
	+							
	−				4			
1-3	G	11	28	24	30			
	M		1	1	1			
	+							
	−		11	4	30			
1-4	G		8	25	8			
	M		2					
	+							
	−							
1-5	G	9	1	11				
	M		1					
	+		1	6				
	−			2				
1-6	G	46	49	33	8			
	M	18						
	+	54	43	30	7			
	−	11	13	6				
1-7	G	16		2	4			
	M			4	1			
	+				8			
	−							
2-3	G						5	
	M							
	+							
	−						5	
2-7	G	2		2				
	M							
	+							
	−							
2-8	G				4	2	8	1
	M			1		1		
	+							
	−					2	6	
2-9	G			3	4	9	2	
	M				3	3		
	+					5		
	−							
2-10	G				1			
	M							
	+							
	−							
3-4	G	2		1	5	6		
	M	1						
	+							
	−					8		

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BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1,1965						
		2170	2200	2230	2260	2290	2320	2340
3-10	G	6		3	7	16		6
	M				2	1		
	+							
	−			2	3			
3-11	G			5	5			2
	M				5			
	+							
	−				4			
4-5	G	13	24	15	1			
	M			1				
	+							
	−	11	17	15				
4-11	G	3				4	20	2
	M	3					4	
	+							
	−			1			21	
4-12	G	2		4	17	2		
	M			1	3			
	+							
	−				14			
4-13	G	3		3	7			
	M	1		1	6			
	+							
	−			1	10			
5-6	G	13	23	33	16			
	M							
	+							
	−	10	22	39	21			
5-13	G	1		4	4			
	M							
	+							
	−				2			
5-14	G			6	13			
	M				3			
	+							
	−				7			
5-15	G			5	12	5		
	M			3	5			
	+							
	−				14			
6-7	G				3	13	1	
	M				1	7	8	
	+							
	−				12		2	
6-15	G	6		6		1		
	M			3				
	+	2		4				
	−	3				2		

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BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965															
		2170	2260	2230	2260	2290	2320	2340									
6-16	G	10		6	15												
	M				1												
	+																
	−				12												
7-8	G			5			5										
	M				2												
	+																
	−																
7-16	G	3															
	M																
	+																
	−																
7-17	G	6			8		14	6									
	M				1												
	+						5										
	−	1			3		1										
8-9	G		6			2	2	2									
	M						1	2									
	+																
	−					2	1	3									
8-17	G	1															
	M																
	+																
	−	1																
8-18	G	1		10		2	1										
	M		2	2			1										
	+			1													
	−	1				2											
8-19	G			1		4	3										
	M		4			3											
	+					5											
	−						1										
9-10	G			2		2	4										
	M																
	+			2			1										
	−					2	2										
9-19	G	1	6	6	3	6											
	M		2	2	3	2											
	+																
	−			1	1												
9-20	G	1	1	1		4											
	M																
	+	1				5											
	−																
10-11	G																
	M																
	+																
	−																

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BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965						
		2170	2200	2230	2260	2290	2320	2340
10-20	G	6		6	4	10	3	
	M			2	3			
	+							
	−					7		
10-21	G	5		2		3	3	
	M							
	+					3		
	−						1	
11-12	G	1					1	
	M							
	+							
	−						1	
11-21	G	2		3	5	7	8	
	M			3	3		2	
	+					10	2	
	−							
11-22	G	3		2		5	1	
	M					1		
	+					4		
	−							
12-13	G						2	
	M							
	+							
	−						2	
12-22	G	5		4	2	6	5	
	M				2		1	
	+					7		
	−							
12-23	G	7		5	1	4	1	
	M	2		1	1	4	2	
	+	1			2		1	
	−							
13-14	G				1		2	
	M						1	
	+				1		1	
	−							
13-23	G			3	6	9	3	1
	M			3		2	1	
	+					5		
	−						1	
13-24	G				2		1	
	M				4			
	+							
	−							
14-15	G			1		2		
	M							
	+							
	−					2		

BASELINES		2170	2200	2230	2260	2290	2320	2340
END OF TIME INTERVAL IN DAYS FROM JAN 1,1965								
14-24	G	3		2	5	10	7	1
	M				2	6	3	1
	+							
	−				4			
14-25	G					2	6	2
	M							
	+					2	2	
	−							
15-16	G			1				
	M							
	+							
	−							
15-25	G	2		1	5	2	14	
	M			1	1			
	+						8	
	−			1	4		3	
15-26	G			3	1	2		
	M							
	+							
	−							
16-17	G			2		2	5	
	M							
	+					1		
	−						2	
16-26	G	6			7	1	19	3
	M					2	10	
	+						2	
	−	1			5			
17-18	G	13		9	1	5	11	
	M	1						
	+	1				6	5	
	−	2		4			4	
17-26	G							
	M							
	+							
	−							
17-27	G	2	2	11		5	1	
	M							
	+					1		
	−							
18-19	G	5	8			1		
	M							
	+	5	6			1		
	−		2					
18-27	G	1	3					
	M							
	+							
	−	1	1					

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BASELINES		2170	2200	2230	2260	2290	2320	2340
	END OF TIME INTERVAL IN DAYS FROM JAN 1,1965							
18-28	G						9	
	M					2	4	
	+						3	
	−							
19-20	G	2						
	M							
	+	3						
	−							
19-28	G						5	
	M							
	+						3	
	−							
19-29	G						9	
	M						5	
	+							
	−							
20-21	G		4	9				
	M					2		
	+			6				
	−							
20-29	G							
	M						3	
	+							
	−							
20-30	G	4					3	1
	M	4					1	
	+							
	−							
21-22	G	2						
	M	1						
	+	1						
	−							
21-30	G	9	14	3		12	6	
	M					4		
	+	2				19		
	−		2	2				
22-23	G	3		2				
	M						3	
	+	3						
	−							
22-30	G		5	10				
	M			5				
	+							
	−		2	2				
22-31	G	15	10			8	4	
	M	1				3		
	+	7				6		
	−		1					

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BASELINES		2170	2200	2230	2260	2290	2320	2340
END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965								
23-24	G		2					2
	M							
	+							
	−							
23-31	G		6	1			4	
	M			11				
	+							
	−							
23-32	G	15	9	1		6	2	
	M	1						
	+	13				5		
	−						1	
24-25	G		4	1		2		1
	M			1				
	+			1		1		
	−					2		
24-32	G	3	9	13	2	5	4	
	M			6	4	1		
	+			3	1	6		
	−							
24-33	G	6				5		
	M		1			2		
	+	4				1		
	−							
25-26	G							
	M							
	+							
	−							
25-33	G	1	7	8		2	4	
	M	1				1		
	+					2		
	−							
25-34	G	7		2		3	1	
	M	3		3				
	+					3		
	−							
26-27	G	6	1	4		6	4	
	M	2				2		
	+					6	2	
	−			1				
26-34	G	1	1	11	3	3		
	M			1	2	3		
	+			1	1			
	−	.						
27-28	G					1	5	
	M					5	6	
	+							
	−							

BASELINES		2170	2200	2230	2260	2290	2320	2340
END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965								
27-34	G		4					
	M							
	+							
	−		3					
27-35	G							
	M							
	+							
	−							
28-29	G					15		24
	M					1		1
	+					20		23
	−							3
28-35	G							12
	M							
	+							4
	−							
29-30	G							1
	M							
	+							
	−							
29-35	G							1
	M							
	+							1
	−							
29-36	G							2
	M							
	+							2
	−							
30-31	G							
	M							
	+							
	−							
30-36	G						3	3
	M							
	+							
	−							
31-32	G		1					
	M							
	+		1					
	−							
31-36	G					4	24	17
	M					5	1	3
	+					8	12	6
	−						2	
32-33	G	1					4	
	M							
	+	1					4	
	−						1	

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BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																
		2170	2200	2230	2260	2290	2320	2340									
32-36	G						8	1									
	M																
	+																
	−																
33-34	G	4	8	9													
	M	4	9	1													
	+		1	8													
	−			1													
34-35	G					1											
	M					2											
	+																
	−																
35-33	G					1	17	2									
	M					4	4	3									
	+					2	6										
	−																
35-36	G			2	1	5	2	23									
	M							10									
	+						2	23									
	−																
36-33	G																
	M																
	+																
	−																
	G																
	M																
	+																
	−																

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TRIANGLES AND BASELINES		2170	2200	2230	2260	2290	2320	2340
1 2 3	G	3		17	5			
	M	5		2	1			
	+							
	−			6	7			
	+	3		6				
	−							
	+			3	4			
	−	3		20	4 1			
1 3 4	G		1	7				
	M		9	1				
	+							
	−		2	13				
	+			1				
	−							
	+							
	−		1	7				
1 4 5	G							
	M							
	+							
	−							
	+							
	−							
	+							
	−							
1 5 6	G		25	10				
	M							
	+		20	5				
	−		16	4				
	+		30	10				
	−			5				
	+							
	−		27	16				
1 6 7	G			9				
	M			12				
	+			9				
	−			9				
	+			2				
	−							
	+							
	−			6				
1 7 2	G	4		8	4			
	M			1				
	+							
	−			2	3			
	+			8	1			
	−							
	+			9	4			
	−			3	1			

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																
		2170	2200	2230	2260	2290	2320	2340										
2 7 8	G			1			3	3										
	M			3	3		3	3										
	+			1			3	3										
	−			1			3	3										
	+			1			5											
	−																	
	+																	
	−							4										
2 8 9	G	4			1		15	8										
	M	2		1	2		5	2										
	+																	
	−				1		4	2										
	+						10											
	−																	
	+	4					8											
	−	4			1		15	14										
2 9 10	G						3	5										
	M			4				1										
	+																	
	−						3	3										
	+							1										
	−																	
	+																	
	−						3	7										
3 2 10	G			1	3		4	8										
	M			2	3		1	3										
	+			1	3		4	8										
	−			1	3		4	7										
	+			2			8	6										
	−																	
	+																	
	−				1			4										
3 10 11	G						3											
	M			2			1											
	+						4											
	−																	
	+						3											
	−																	
	+																	
	−						2											
4 3 11	G																	
	M																	
	+																	
	−																	
	+																	
	−																	
	+																	
	−																	

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TRIANGLES AND BASELINES		2170	2200	2230	2260	2290	2320	2340
4 11 12	G			3			7	6
	M			2			1	1
	+							
	−						3	2
	+			5			6	7
	−							
	+			3			5	
	−			3			7	6
4 12 13	G	3		3				
	M	1						
	+							
	−	3						
	+	2						
	−							
	+							
	−	3						
5 4 13	G	4		6	4			
	M			3				
	+			5				
	−			6				
	+			1				
	−							
	+							
	−			8				
5 13 14	G	5		4	3			
	M							
	+							
	−			3	4			
	+							
	−							
	+							
	−			4	4			
5 14 15	G			3	4			
	M			1	3			
	+							
	−				5			
	+			2	3			
	−							
	+							
	−			3	4			
6 5 15	G			16	4			
	M			5				
	+			12				
	−			16				
	+			13				
	−							
	+							
	−			10				

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TRIANGLES AND BASELINES		2170	2200	2230	2260	2290	2320	2340
6 / 15 / 16	G			6	5			
	M							
	+							
	−			2	6			
	+			7	1			
	−							
	+			4	1			
	−			6	5			
6 / 16 / 7	G			7	1			
	M							
	+							
	−			11				
	+			3				
	−							
	+			7				
	−			3				
7 / 16 / 17	G			3	1	9	3	
	M			3		1	1	
	+			3		9	2	
	−			3		6	3	
	+			2		10		
	−							
	+							
	−			3		6	4	
7 / 17 / 18	G			1		1	2	
	M						1	
	+							
	−					2	2	
	+					1	1	
	−						2	
	+							
	−							
8 / 17 / 18	G	3		1		1		
	M							
	+							
	−	1						
	+	4						
	−							
	+							
	−	4						
8 / 18 / 19	G	7	8	2	1		1	
	M	2			2			
	+							
	−	5						
	+	5			2		2	
	−							
	+	4						
	−	6					1	

TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965						
		2170	2200	2230	2260	2290	2320	2340
9 8 19	G	13	4	4		1	9	
	M	1		5		1	2	
	+	19		3		1	9	
	−	7		2			8	
	+	4		2		2	5	
	−							
	+							
	−	3		1			3	
9 19 20	G	6	6	2			1	
	M	1	6	4				
	+		9	3				
	−	1						
	+	8	2	1			2	
	−							
	+	4	6	2				
	−	4	6	2				
10 9 20	G	6		1		1	4	
	M	3		2		1		
	+	8		2			3	
	−					1	4	
	+	4				2	5	
	−							
	+							
	−	1		1			1	
10 20 21	G	3		1			6	
	M	4						
	+							
	−						3	
	+	4					5	
	−							
	+						1	
	−	3					6	
11 10 21	G			2			3	
	M						1	
	+						1	
	−							
	+						4	
	−							
	+							
	−						3	
11 21 22	G			5			6	
	M						2	
	+						5	
	−							
	+						4	
	−							
	+						4	
	−						6	

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																
		2170	2200	2230	2260	2290	2320	2340										
12 11 22	G M + −	4		5			6 2 5 6											
	+ − + −						4 — 4											
12 22 23	G M + −			1			6 — 5											
	+ − + −						6 3 5											
13 12 23	G M + −	2		3			10 4 8 7	5 3 4										
	+ − + −						6 — 8	3 — 5										
13 23 24	G M + −						7 2 3											
	+ − + −						5 1 7											
14 13 24	G M + −	3 3 3 2		1			6 2 6 6	7 3 5 7										
	+ − + −	5					12 — 3	7 — 3										
14 24 25	G M + −	1 2		5 1 4		2	6 — 5	2 3										
	+ − + −	2		4 3 4		2 1 2	2 5	4										

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Table: END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965

TRIANGLES AND BASELINES		2170	2200	2230	2260	2290	2320	2340
15 / 14 / 25	G	4		1	6		13	9
	M				1		8	1
	+	4			5			9
	−			1	4		13	5
	+			2	1		10	2
	−							
	+							
	−	5			7		8	12
15 / 25 / 26	G			2			1	2
	M							2
	+							
	−						2	1
	+							
	−							
	+							
	−						1	2
15 / 26 / 16	G			3	1		8	8
	M			1	3			3
	+							
	−				1		1	2
	+			3	1		6	6
	−			3	1		8	8
	+			4	1		10	9
	−							
16 / 26 / 17	G			1			2	3
	M			1				2
	+							
	−			1			3	1
	+			1			1	
	−							
	+			1			1	3
	−							
17 / 26 / 27	G	3		2				
	M	2						
	+							
	−	2						
	+	1						
	−							
	+							
	−	4						
17 / 27 / 18	G	3	5	3			1	
	M	2						
	+							
	−							
	+	5						
	−							
	+	3						
	−	2						

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965															
		2170	2200	2230	2260	2290	2320	2340									
18 27 28	G M + −																
	+ − + −																
19 18 28	G M + −																
	+ − + −																
19 28 29	G M + −						1 8										
	+ − + −						1 1										
20 19 29	G M + −						6										
	+ − + −																
20 21 30	G M + −	10 3 8 2	7 7				1 3 1										
	+ − + −	8 2 19	5 7				2										
21 22 30	G M + −	7 6	8 7	3 1 2 2			7 1 4 6										
	+ − + −	8 4	12 3	5 1			9 5										

TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965										
		2170	2200	2230	2260	2290	2320	2340				
22 23 31	G M + −	2 1 1	2 4 2				3 2 3 3					
	+ − + −	2 1	3				4					
22 30 31	G M + −	4	3	3			2 3					
	+ − + −	7 4 3					3 2 2					
23 24 32	G M + −	4 4	1 2 1				2 1 1 2					
	+ − + −	3 4					4					
23 31 32	G M + −		1 5				1					
	+ − + −		1 ... 1									
24 32 33	G M + −	5 6 4	3 11 5				1 3					
	+ − + −	1 5 5	3 3				1 1					
25 24 33	G M + −	3 4 3	2	2		1						
	+ − + −	1										

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TRIANGLES AND BASELINES		2170	2200	2230	2260	2290	2320	2340
25 / 33 / 34	G	5	10				6	
	M	4					2	
	+							
	–							
	+	4					4	
	–							
	+	2						
	–	5					6	
26 / 25 / 34	G	4		2				
	M	1						
	+	3						
	–							
	+	6						
	–							
	+							
	–	2						
26 / 34 / 27	G	9	17	3			9	
	M	4		2			3	
	+							
	–	6		2			5	
	+	5					5	
	–							
	+	9		3			9	
	–	9		1			8	
27 / 34 / 35	G							
	M						1	
	+							
	–							
	+							
	–							
	+							
	–							
27 / 35 / 28	G							
	M							
	+							
	–							
	+							
	–							
	+							
	–							
28 / 35 / 29	G						6	
	M						1	
	+						6	
	–							
	+						10	
	–							
	+						2	
	–							

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TRIANGLES AND BASELINES		2170	2200	2230	2260	2290	2320	2340									
END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
29 30 36	G M + −							4									
	+ − + −																
30 20 29	G M + −																
	+ − + −																
31 30 36	G M + −						10	8									
	+ − + −																
32 31 36	G M + −						6 1 2 1										
	+ − + −						5 7										
33 36 35	G M + −																
	+ − + −																
34 33 35	G M + −						3 6 3										
	+ − + −						4										

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TRIANGLES AND BASELINES		END OF TIME INTERVAL IN DAYS FROM JAN 1, 1965																	
		2170	2200	2230	2260	2290	2320	2340											
35 36 29	G M + −																		
	+ − + −																		
36 32 33	G M + −						1												
	+ − + −																		
	G M + −																		
	+ − + −																		
	G M + −																		
	+ − + −																		
	G M + −																		
	+ − + −																		
	G M + −																		
	+ − + −																		

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APPENDIX XI
RELATION BETWEEN LOCAL TIME AND LAUNCH RIGHT ASCENSION

The solar time at a point on the earth's surface is the angle from the plane containing the sun and the earth's axis, eastward to the meridian plane of the point; it is a.m. if the angle is measured from the shadow side of the sun plane and p.m. if measured from the bright side. This relationship is illustrated in figure 50. The local standard time depends on the longitude difference between the point and the standard time meridian for the applicable time zone; for Pacific Standard Time, this meridian is 120 degrees West and the local solar time is 4 minutes slower than Pacific Standard Time for each additional degree of westward longitude.
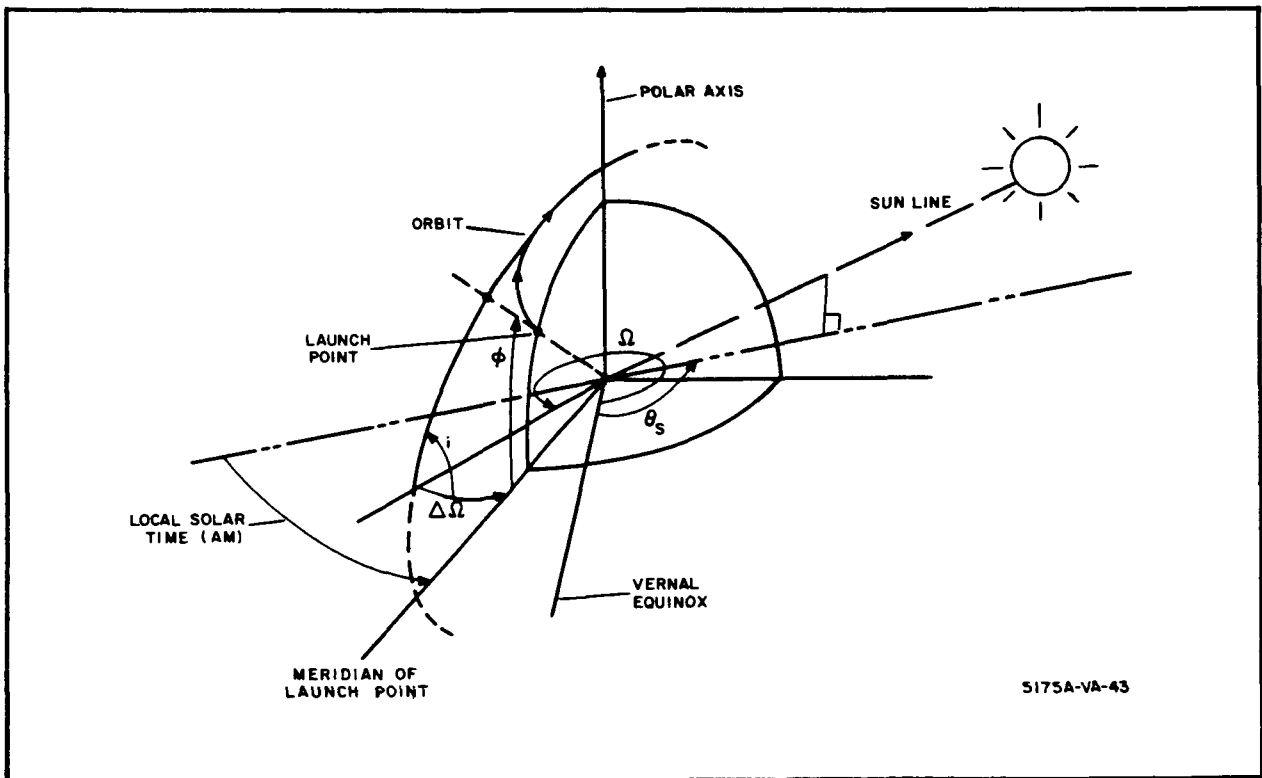
Figure 50. Local Time/Launch Right Ascension Geometry

To obtain a launch right ascension of Ω , the right ascension of the launch point is approximately $\Omega + \Delta\Omega$, where $\Delta\Omega \approx (\frac{\pi}{2} - i)\tan\phi$ for highly inclined orbits ($i \geq 85°$). The local solar time is then ($\Omega + \Delta\Omega - \theta_s - 180°$) for the geometry illustrated. For launch from Vandenburg AFB ($\phi \approx 34°$ N, $\lambda \approx 120.6°$ W) at an inclination of 87 degrees, $\Delta\Omega \approx 2.0°$ and local solar time lags Pacific Standard Time by 2.4 minutes. For a 1 June launch with $\Omega = 345°$, the Pacific Standard Time at launch is thus approximately

$\quad$ PST ≈ 2.4 minutes $+ 1/15$ $(345 + 2 - \frac{71}{91} \times 90 - 180)$ hours

$\quad$ PST ≈ 2.4 minutes $+ 6.46$ hours

$\quad$ PST $\approx 6{:}30.0$ a. m.

Similarly, the approximate launch times of other orbits can be found.

APPENDIX XII

LATITUDE AND LONGITUDE OF SATELLITE

Let a terrestial coordinate system be defined as shown in figure 51, i. e., with the x axis in the plane of the equator and through the Greenwich meridian, the z axis through the North Pole, and the y axis normal in a right-handed sense. Then the orbit of the satellite can be expressed by the classical orbital elements representation in this system just as in the geocentric system. The one exception is that the angle of right ascension crossing, Ω_T, is referenced to the Greenwich meridian rather than the First Point of Aries. The angles i, ω, and u are defined in the standard sense, i. e., inclination of the orbital plane, argument of perigee, and true anomaly of the satellite.
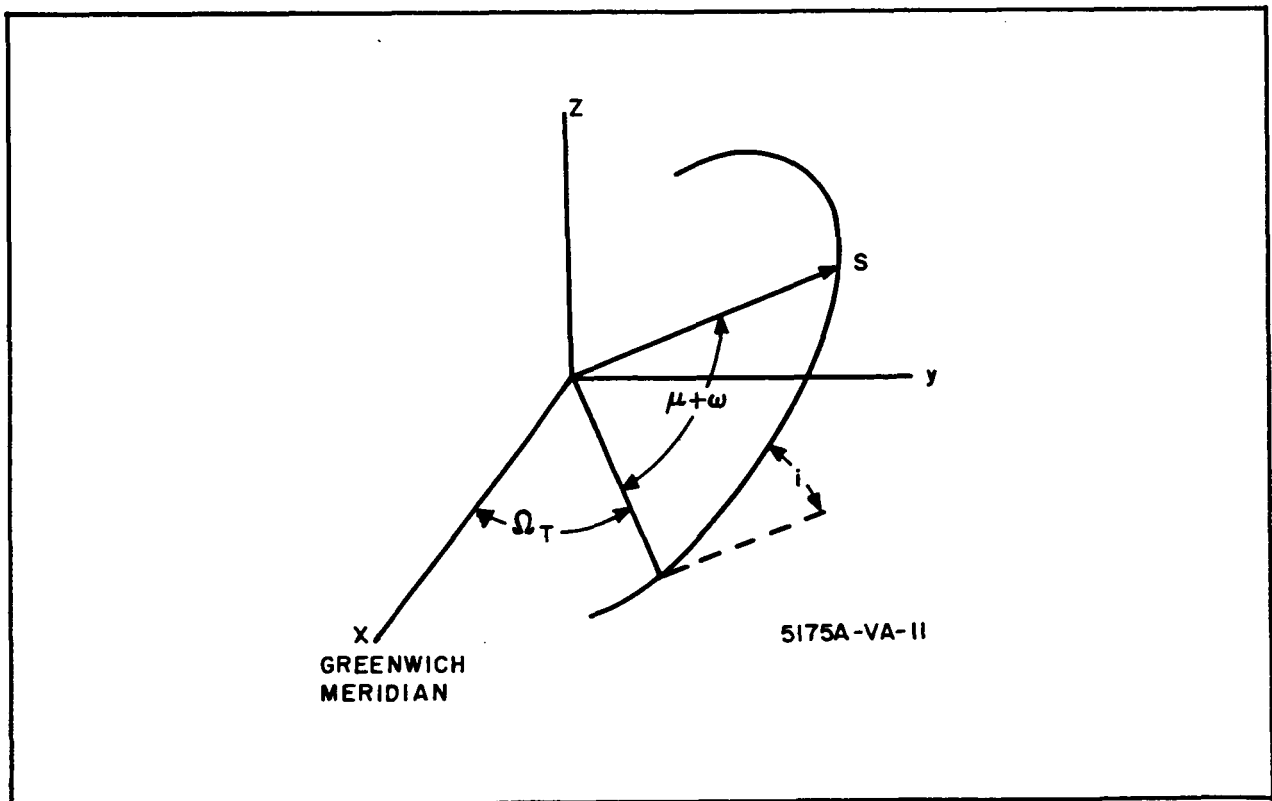
Figure 51. Geometry of Orbit

By the laws of Keplerian mechanics, the radius vector, $\bar{r}$, to the satellite is defined as:

$$\bar{r} = r \begin{bmatrix} \cos(u+\omega)\cos\Omega_T - \sin(u+\omega)\cos i\sin\Omega_T \\ \cos(u+\omega)\sin\Omega_T + \sin(u+\omega)\cos i\cos\Omega_T \\ \sin(u+\omega)\sin i \end{bmatrix}$$

where

$$r = a(1 - e\cos E)$$

and a and E are the semimajor axis and eccentric anomaly of the satellite respectively. Then the latitude, ϕ, and longitude, θ, of the satellite can be defined by the angle between the radius vector and the plane of the equator, and the angle between the projection of the radius vector in the plane of the equator and the x axis or:

$$\theta = \tan^{-1} \left[\frac{\cos(u+\omega)\sin\Omega_T + \sin(u+\omega)\cos i\cos\Omega_T}{\cos(u+\omega)\cos\Omega_T - \sin(u+\omega)\cos i\sin\Omega_T} \right]$$

$$\phi = \tan^{-1} \left[\frac{\sin(u+\omega)\sin i}{(\cos^2(u+\omega) + \sin^2(u+\omega)\cos^2 i)^{1/2}} \right]$$